Section 3: The product and quotient rules

Section test

1. The product rule says that if $y = u \times v$, then

(a) $\frac{dy}{dx} = \frac{du}{dx} \times \frac{dv}{dx}$ (b) $\frac{dy}{dx} = u\frac{du}{dx} + v\frac{dv}{dx}$

(c) $\frac{dy}{dx} = u\frac{dv}{dx} - v\frac{du}{dx}$ (d) $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

2. The quotient rule says that if $y = \frac{u}{v}$, then

(a) $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ (b) $\frac{dy}{dx} = \frac{v\frac{du}{dx} + u\frac{dv}{dx}}{v^2}$

(c) $\frac{dy}{dx} = \frac{u\frac{dv}{dx} - v\frac{du}{dx}}{v^2}$ (d) $\frac{dy}{dx} = \frac{du/dx}{dv/dx}$

3. The derivative of $x^{\frac{1}{3}}(1-3x)^4$ is:

(a) $\frac{3}{5}x^{\frac{5}{3}}(1-3x)^4 - 12x^{\frac{1}{3}}(1-3x)^3$ (b) $\frac{1}{3}x^{-\frac{2}{3}}(1-3x)^4 + 4x^{\frac{1}{3}}(1-3x)^3$

(c) $\frac{1}{3}x^{-\frac{2}{3}}(1-3x)^4 - 12x^{\frac{1}{3}}(1-3x)^3$ (d) $\frac{1}{3}x^{-\frac{2}{3}}(1-3x)^4 + 12x^{\frac{1}{3}}(1-3x)^3$

4. The derivative of $x^2(2x+1)^4$ is:

(a) $2x(2x+1)^3(4x+1)$ (b) $16x(2x+1)^3$

(c) $8x(2x+1)^3$ (d) $2x(2x+1)^3(6x+1)$

5. The derivative of $x\sqrt{2x-3}$ is:

(a) $\frac{1}{\sqrt{2x-3}}$ (b) $\frac{1}{2\sqrt{2x-3}}$

(c) $\frac{3(x-1)}{\sqrt{2x-3}}$ (d) $\frac{5x-6}{2\sqrt{2x-3}}$

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6. The derivative of $\frac{x^2}{1-2x}$ is:

(a) $\frac{2x(1-x)}{(1-2x)^2}$ (b) $\frac{2x(x-1)}{(1-2x)^2}$

(c) $-x$ (d) $\frac{2x(1-3x)}{(1-2x)^2}$

7. Given that $y = \frac{\sqrt{x}}{1+\sqrt{x}}$, $\frac{dy}{dx} =$

(a) 1 (b) $\frac{1}{2\sqrt{x}(1+x)}$

(c) $\frac{1}{2\sqrt{x}(1+\sqrt{x})^2}$ (d) $\frac{1+2\sqrt{x}}{2\sqrt{x}(1+\sqrt{x})^2}$

8. Find the gradient of the curve $y = \sqrt{x}(2-x)^3$ at the point where $x = 4$.

9. Find the gradient of the curve $y = \frac{x^2+2}{1-3x}$ at the point where $x = -1$.

10. Find the turning points on the curve $y = \frac{3+x^2}{1-x}$, giving their nature.

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Section test Solutions

1) By definition: $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

2) By definition: $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

3) $u = x^{\frac{1}{3}} \quad \Rightarrow \frac{du}{dx} = \frac{1}{3}x^{-\frac{2}{3}}$

$v = (1-3x)^4 \quad \Rightarrow \frac{dv}{dx} = 4(1-3x)^3 \times -3 = -12(1-3x)^3$

Using the product rule: $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

$= x^{\frac{1}{3}}\left(-12(1-3x)^3\right) + (1-3x)^4 \times \frac{1}{3}x^{-\frac{2}{3}}$

$= -12x^{\frac{1}{3}}(1-3x)^3 + \frac{1}{3}x^{-\frac{2}{3}}(1-3x)^4$

4) $u = x^2 \Rightarrow \frac{du}{dx} = 2x$

$v = (2x+1)^4 \Rightarrow \frac{dv}{dx} = 8(2x+1)^3$

Using the product rule: $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

$= x^2\left(8(2x+1)^3\right) + (2x+1)^4(2x)$

$= 8x^2(2x+1)^3 + 2x(2x+1)^4$

$= 2x(2x+1)^3(4x+2x+1)$

$= 2x(2x+1)^3(6x+1)$

5) $u = (2x-3)^{\frac{1}{2}} \Rightarrow \frac{du}{dx} = (2x-3)^{-\frac{1}{2}}$

$v = x \Rightarrow \frac{dv}{dx} = 1$

Using the product rule: $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

$$= (2x-3)^{\frac{1}{2}}(1) + (x)(2x-3)^{-\frac{1}{2}}$$
$$= \sqrt{2x-3} + \frac{x}{\sqrt{2x-3}}$$
$$= \frac{2x-3+x}{\sqrt{2x-3}}$$
$$= \frac{3x-3}{\sqrt{2x-3}}$$
$$= \frac{3(x-1)}{\sqrt{2x-3}}$$

6) $u = x^2 \Rightarrow \frac{du}{dx} = 2x$

$v = 1-2x \Rightarrow \frac{dv}{dx} = -2$

Using the quotient rule: $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$$= \frac{(1-2x)(2x) - \left(x^2\right)(-2)}{(1-2x)^2}$$
$$= \frac{2x - 4x^2 + 2x^2}{(1-2x)^2}$$
$$= \frac{2x(1-x)}{(1-2x)^2}$$

7) $u = \sqrt{x} = x^{\frac{1}{2}} \Rightarrow \frac{du}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$

$v = 1+\sqrt{x} = 1+x^{\frac{1}{2}} \Rightarrow \frac{dv}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$

Using the quotient rule: $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$$= \frac{\left(1+x^{\frac{1}{2}}\right)\left(\frac{1}{2}x^{-\frac{1}{2}}\right) - \left(x^{\frac{1}{2}}\right)\left(\frac{1}{2}x^{-\frac{1}{2}}\right)}{\left(1+x^{\frac{1}{2}}\right)^2}$$
$$= \frac{\left(\frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{2}\right) - \frac{1}{2}}{\left(1+x^{\frac{1}{2}}\right)^2}$$
$$= \frac{\frac{1}{2}x^{-\frac{1}{2}}}{\left(1+x^{\frac{1}{2}}\right)^2} = \frac{1}{2\sqrt{x}\left(1+\sqrt{x}\right)^2}$$

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8) $y = \sqrt{x}(2-x)^3$

$u = \sqrt{x} = x^{\frac{1}{2}} \quad \Rightarrow \frac{du}{dx} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

$v = (2-x)^3 \quad \Rightarrow \frac{dv}{dx} = 3(2-x)^2 \times -1 = -3(2-x)^2$

Using the product rule: $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

$$= \sqrt{x} \times -3(2-x)^2 + (2-x)^3 \times \frac{1}{2\sqrt{x}}$$

$$= -3\sqrt{x}(2-x)^2 + \frac{(2-x)^3}{2\sqrt{x}}$$

When $x = 4$, $\frac{dy}{dx} = -3 \times 2(2-4)^2 + \frac{(2-4)^3}{2\times 2} = -24 - 2 = -26$

9) $y = \frac{x^2+2}{1-3x}$

Let $u = x^2 + 2 \quad \Rightarrow \frac{du}{dx} = 2x$

Let $v = 1 - 3x \quad \Rightarrow \frac{dv}{dx} = -3$

Using the quotient rule: $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$$= \frac{2x(1-3x) - (x^2+2)\times -3}{(1-3x)^2}$$

$$= \frac{2x(1-3x) + 3(x^2+2)}{(1-3x)^2}$$

When $x = -1$, $\frac{dy}{dx} = \frac{-2(1+3) + 3(1+2)}{(1+3)^2} = \frac{-8+9}{16} = \frac{1}{16}$

10) $y = \frac{3+x^2}{1-x}$

$u = 3 + x^2 \Rightarrow \frac{du}{dx} = 2x$

$v = 1 - x \Rightarrow \frac{dv}{dx} = -1$

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Using the quotient rule: $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$$= \frac{(1-x)(2x) - \left(3+x^2\right)(-1)}{(1-x)^2}$$

$$= \frac{2x - 2x^2 + 3 + x^2}{(1-x)^2}$$

$$= \frac{2x - x^2 + 3}{(1-x)^2}$$

At turning points, $\frac{dy}{dx} = 0$

$$2x - x^2 + 3 = 0$$

$$(-x+3)(x+1) = 0$$

$$x = -1 \text{ or } 3$$

For $x < -1$, $\frac{dy}{dx} < 0$

For $-1 < x < 3$, $\frac{dy}{dx} > 0$

For $x > 3$, $\frac{dy}{dx} < 0$

So there is a minimum point where x = -1 and a maximum point where x = 3.

When $x = -1$, $y = \frac{3+(-1)^2}{1-(-1)} = 2$

When $x = 3$, $y = \frac{3+(3)^2}{1-(3)} = -6$

So the turning points are a minimum at (-1, 2) and a maximum at (3, -6).