Section 3: The product and quotient rules

Crucial points

1. **Make sure you use the product rule correctly**

   Example: Differentiate $y = x\sqrt{(1+x)},\ u = x,\ v = \sqrt{(1+x)}$

   ✗ ***Wrong*** $\frac{dy}{dx} = \frac{du}{dx} \times \frac{dv}{dx} = 1 \times \frac{1}{2}(1+x)^{-\frac{1}{2}}$

   ✓ ***Right*** $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx} = x \times \frac{1}{2}(1+x)^{-\frac{1}{2}} + 1 \times (1+x)^{\frac{1}{2}}$

2. **Don't mix up the chain rule and the product rule**

   Functions like y = sin 2x, y = ln(1 + x^2) – use the chain rule

   Only use the product rule when the function clearly splits up into a product

3. **Make sure you use the quotient rule correctly**

   Don't get 'u' and 'v' mixed up

   Remember the negative sign in the numerator

   $y = \frac{u}{v}$ ✗ ***Wrong***: $\frac{dy}{dx} = \frac{v\frac{du}{dx} + u\frac{dv}{dx}}{v^2}$ $\quad \frac{dy}{dx} = \frac{u\frac{dv}{dx} - v\frac{du}{dx}}{v^2}$

   ✓ ***Right*** $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

4. **Be careful when finding stationary points of quotient functions**

   Once you have found $\frac{dy}{dx}$, to find the stationary point you solve $\frac{dy}{dx}$ = 0. This is true when the numerator of the fraction equals zero, NOT the denominator, OR when numerator = denominator!

   Example: Find the turning points of $y = \frac{x^2}{1-x}$

   First use the quotient rule to differentiate:

   $$y = \frac{x^2}{1-x}, \Rightarrow \frac{dy}{dx} = \frac{(1-x)2x - x^2(-1)}{(1-x)^2} = \frac{2x - x^2}{(1-x)^2}$$

   ✗ ***Wrong***: $\frac{dy}{dx} = 0$ when $2x - x^2 = (1-x)^2$, etc

   or $\frac{dy}{dx} = 0$ when $2x - x^2 = 0$ or $(1-x)^2$, etc

   ✓ ***Right*** $\frac{dy}{dx} = 0$ when $2x - x^2 = 0, \Rightarrow x(2-x) = 0$

   $\Rightarrow x = 0, y = 0,$ or $x = 2, y = -4$