Section 1: Differentiating exponentials and logarithms

Section test

1. The derivative of e^{-3x} is

(a) $-3e^{-2x}$

(b) $\frac{e^{-3x}}{-3}$

(c) $-3e^{-3x}$

(d) $\frac{e^{-4x}}{4}$

2. The derivative of $\ln(1-x)$ is

(a) $-\frac{1}{x}$

(b) $\frac{1}{x-1}$

(c) $\frac{1}{1-x}$

(d) $\ln(-1)$

3. The derivative of 5^x is

(a) 4^x

(b) $x5^{x-1}$

(c) $5^x \ln 5$

(d) $5^x \ln x$

4. Find the gradient of the curve $y = \ln(2-3x)$ at the point with x-coordinate 0.

5. The gradient of the curve $y = e^{3x-1}\ln 2x$, at the point with x-coordinate 1 is

(a) $e^2(\frac{1}{2} + \ln 2)$

(b) $e^2(\frac{1}{2} + 3\ln 2)$

(c) $e^2(1 + \ln 2)$

(d) $e^2(1 + 3\ln 2)$

6. The derivative of $\frac{\ln x}{1+x}$ is:

(a) $\frac{1+x-\ln x}{x(1+x)^2}$

(b) $\frac{1+x-x\ln x}{x(1+x)^2}$

(c) $\frac{1+x+x\ln x}{x(1+x)^2}$

(d) $\frac{1+x-x\ln x}{(1+x)^2}$

7. The value(s) of x for which the gradient of the curve $y = \frac{1}{1 - \ln x}$ is zero are

(a) $x = 0$ and $x = 1$
(b) $x = 1$
(c) There are none
(d) $x = 0$

8. Find the equation of the tangent to the curve $y = e^{2x-1}$ at the point where $x = 1$.

(a) $y = 2ex - e$
(b) $y = 2ex - 1$
(c) $y = 2x - e$
(d) $y = 2x - 1$

9. The tangent to the curve $y = 2^x$ at the point where $x = 3$ cuts the y-axis at the point

(a) $(0, 8 - 8\ln 2)$
(b) $(0, 24\ln 2)$
(c) $(0, 8 - 24\ln 2)$
(d) $(0, 8\ln 2)$

10. Find the turning point of the curve $y = x - \ln 2x$ and state its nature.

Solutions to section test

1. $y = e^{-3x}$

   Let $u = -3x \quad \Rightarrow \dfrac{du}{dx} = -3$

   $y = e^{u} \quad \Rightarrow \dfrac{dy}{du} = e^{u}$

   Using the chain rule, $\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx} = e^{u} \times -3 = -3e^{-3x}$

2. $y = \ln(1-x)$

   Let $u = 1-x \quad \Rightarrow \dfrac{du}{dx} = -1$

   $y = \ln u \quad \Rightarrow \dfrac{dy}{du} = \dfrac{1}{u}$

   Using the chain rule, $\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx} = \dfrac{1}{u} \times -1 = -\dfrac{1}{1-x} = \dfrac{1}{x-1}$

3. $y = 5^{x} = e^{\ln 5^{x}} = e^{x\ln 5}$

   $\dfrac{dy}{dx} = \ln 5 \times e^{x\ln 5} = 5^{x} \ln 5$

4. $y = \ln(2-3x) \Rightarrow \dfrac{dy}{dx} = \dfrac{-3}{2-3x}$

   When $x = 0$, gradient $= \dfrac{-3}{2} = -\dfrac{3}{2}$

5. $y = e^{3x-1} \ln 2x$

   Let $u = e^{3x-1} \quad \Rightarrow \dfrac{du}{dx} = 3e^{3x-1}$

   Let $v = \ln 2x = \ln 2 + \ln x \quad \Rightarrow \dfrac{dv}{dx} = \dfrac{1}{x}$

   Using the product rule, $\dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$

   $$= e^{3x-1} \times \frac{1}{x} + \ln 2x \times 3e^{3x-1}$$

   $$= e^{3x-1}\left(\frac{1}{x} + 3\ln 2x\right)$$

   When $x = 1$, gradient $= e^{2}(1 + 3\ln 2)$

6. $y = \dfrac{\ln x}{1+x}$

Let $u = \ln x \quad \Rightarrow \dfrac{du}{dx} = \dfrac{1}{x}$

Let $v = 1 + x \quad \Rightarrow \dfrac{dv}{dx} = 1$

Using the quotient rule, $\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

$$= \frac{(1+x)\times\frac{1}{x} - \ln x \times 1}{(1+x)^2}$$

$$= \frac{1 + x - x\ln x}{x(1+x)^2}$$

7. $y = \dfrac{1}{1-\ln x} = (1-\ln x)^{-1}$

Let $u = 1 - \ln x \quad \Rightarrow \dfrac{du}{dx} = -\dfrac{1}{x}$

$y = u^{-1} \quad \Rightarrow \dfrac{dy}{du} = -u^{-2} = -\dfrac{1}{u^2}$

Using the chain rule, $\dfrac{dy}{dx} = \dfrac{dy}{du}\times\dfrac{du}{dx} = -\dfrac{1}{u^2}\times -\dfrac{1}{x} = \dfrac{1}{x(1-\ln x)^2}$

Since the numerator is never zero, there are no stationary points.

8. $y = e^{2x-1}$

$\dfrac{dy}{dx} = 2e^{2x-1}$

When $x = 1$, gradient $= 2e$

When $x = 1$, $y = e$

Equation of tangent is $y - e = 2e(x-1)$

$$y - e = 2ex - 2e$$

$$y = 2ex - e$$

9. $y = 2^x \Rightarrow \dfrac{dy}{dx} = 2^x \ln 2$

When $x = 3$, gradient $= 2^3 \ln 2 = 8\ln 2$

When $x = 3$, $y = 2^3 = 8$

Equation of tangent is $y-8=8\ln 2(x-3)$

$$y-8=8x\ln 2-24\ln 2$$

$$y=8x\ln 2-24\ln 2+8$$

When $x=0$, $y=8-24\ln 2$

10. $y=x-\ln 2x$

$$\frac{dy}{dx}=1-\frac{1}{x}$$

When gradient = 0, $1-\frac{1}{x}=0 \quad \Rightarrow \frac{1}{x}=1 \quad \Rightarrow x=1$

When x = 1, $y=1-\ln 2$.

$$\frac{dy}{dx}=1-x^{-1} \quad \Rightarrow \frac{d^2y}{dx^2}=-1\times -x^{-2}=\frac{1}{x^2}>0$$

so (1, 1 – ln 2) is a minimum point.