Section 2: Differentiating trigonometric functions

Section test

1. The derivative of $\cos 2x$ is

(a) $-\sin 2x$ (b) $-2\sin 2x$
(c) $2\sin 2x$ (d) $\sin 2x$

2. The derivative of $\ln(\sin x)$ is

(a) $\dfrac{\cos x}{\sin x}$ (b) $\dfrac{1}{\sin x}$
(c) $\dfrac{1}{\cos x}$ (d) $-\dfrac{\cos x}{\sin x}$

3. The derivative of $\sin x \cos x$ is

(a) $-\sin x \cos x$ (b) $\cos^2 x + \sin^2 x$
(c) $\cos x \sin x$ (d) $\cos^2 x - \sin^2 x$

4. The derivative of $e^{\cos x}$ is

(a) $e^{-\sin x}$ (b) $\sin x e^{\cos x}$
(c) $-\sin x e^{\cos x}$ (d) $e^{\sin x}$

5. Find the gradient of the curve $y = \tan 2x$ at the point with x-coordinate 0.

6. The derivative of $\sqrt{\sin 2x}$ is

(a) $\dfrac{\cos x}{\sqrt{\sin x}}$ (b) $-\dfrac{\cos 2x}{\sqrt{\sin 2x}}$
(c) $\dfrac{\cos 2x}{\sqrt{\sin 2x}}$ (d) $\dfrac{\cos 2x}{2\sqrt{\sin 2x}}$

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7. Given that $y = \dfrac{1}{\cos 2x}$, then $\dfrac{dy}{dx} =$

(a) $-\dfrac{\tan 2x}{\cos 2x}$

(b) $-\dfrac{2}{\cos^2 2x}$

(c) $\dfrac{\tan 2x}{\cos 2x}$

(d) $\dfrac{2\tan 2x}{\cos 2x}$

8. Find the gradient of the curve $y = \dfrac{1}{1+\cos x}$ at the point $\left(\dfrac{\pi}{2}, 1\right)$.

9. The deviation x cm of a spring at time t seconds is given by $x = 5 \sin 3t$. Find the rate of change of the deviation after 1 second.

10. The gradient of the curve $y = \cos 2x°$ when $x = 15$ is:

(a) $-\dfrac{\pi}{180}$

(b) -1

(c) $-\dfrac{\pi\sqrt{3}}{180}$

(d) 1

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Solutions to section test

1. $y = \cos 2x$

   Using the chain rule, $\dfrac{dy}{dx} = -\sin 2x \times 2 = -2\sin 2x$

2. $y = \ln(\sin x)$

   Let $u = \sin x \quad \Rightarrow \dfrac{du}{dx} = \cos x$

   $y = \ln u \quad \Rightarrow \dfrac{dy}{du} = \dfrac{1}{u}$

   Using the chain rule, $\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx} = \dfrac{1}{u} \times \cos x = \dfrac{\cos x}{\sin x} = \cot x$

3. $y = \sin x \cos x$

   Let $u = \sin x \quad \Rightarrow \dfrac{du}{dx} = \cos x$

   Let $v = \cos x \quad \Rightarrow \dfrac{dv}{dx} = -\sin x$

   Using the product rule, $\dfrac{dy}{dx} = \sin x \times -\sin x + \cos x \times \cos x$

   $= \cos^2 x - \sin^2 x$

4. $y = e^{\cos x}$

   Let $u = \cos x \quad \Rightarrow \dfrac{du}{dx} = -\sin x$

   $y = e^u \quad \Rightarrow \dfrac{dy}{du} = e^u$

   Using the chain rule, $\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx} = e^u \times -\sin x = -\sin x\, e^{\cos x}$

5. $y = \tan 2x = \dfrac{\sin 2x}{\cos 2x}$

   $$\frac{dy}{dx} = \frac{\cos 2x \times 2\cos 2x - \sin 2x \times -2\sin 2x}{\cos^2 2x}$$

   $$= \frac{2\cos^2 2x + 2\sin^2 2x}{\cos^2 2x} = \frac{2}{\cos^2 2x}$$

   When $x = 0$, gradient $= \dfrac{2}{1^2} = 2$

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6. $y = \sqrt{\sin 2x} = (\sin 2x)^{\frac{1}{2}}$

Let $u = \sin 2x \quad \Rightarrow \frac{du}{dx} = 2\cos 2x$

$y = u^{\frac{1}{2}} \quad \Rightarrow \frac{dy}{du} = \tfrac{1}{2}u^{-\frac{1}{2}} = \frac{1}{2\sqrt{u}}$

Using the chain rule: $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \frac{1}{2\sqrt{u}} \times 2\cos 2x = \frac{\cos 2x}{\sqrt{\sin 2x}}$

7. $y = \frac{1}{\cos 2x} = (\cos 2x)^{-1}$

Using the chain rule, $\frac{dy}{dx} = -(\cos 2x)^{-2} \times -2\sin 2x$

$$= \frac{2\sin 2x}{\cos^2 2x}$$

$$= \frac{2\tan 2x}{\cos 2x}$$

8. $y = \frac{1}{1+\cos x} = (1+\cos x)^{-1}$

Let $u = 1+\cos x \quad \Rightarrow \frac{du}{dx} = -\sin x$

$y = u^{-1} \quad \Rightarrow \frac{dy}{du} = -u^{-2} = -\frac{1}{u^2}$

Using the chain rule: $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = -\frac{1}{u^2} \times -\sin x = \frac{\sin x}{(1+\cos x)^2}$

When $x = \frac{\pi}{2}$, gradient $= \frac{\sin\frac{\pi}{2}}{(1+\cos\frac{\pi}{2})^2} = \frac{1}{(1+0)^2} = 1$

9. $x = 5\sin 3t$

$\frac{dx}{dt} = 5 \times 3\cos 3t = 15\cos 3t$

When t = 1, $\frac{dx}{dt} = 15\cos 3 = -14.85$

The rate of change of the deviation is – 14.85 cm / second.

10. $y = \cos 2x^\circ = \cos\left(2 \times \frac{\pi x}{180}\right) = \cos\left(\frac{\pi x}{90}\right)$

$\frac{dy}{dx} = -\sin\left(\frac{\pi x}{90}\right) \times \frac{\pi}{90} = -\frac{\pi}{90}\sin\left(\frac{\pi x}{90}\right)$

When $x = 15$, gradient $= -\dfrac{\pi}{90}\sin\left(\dfrac{\pi \times 15}{90}\right) = -\dfrac{\pi}{90}\sin\left(\dfrac{\pi}{6}\right)$

$$= -\frac{\pi}{90} \times \frac{1}{2} = -\frac{\pi}{180}$$