Section 2: Further trigonometric equations

Section test

1. Express $2\cos\theta - 3\sin\theta$ in the form $r\cos(\theta + \alpha)$, where $r > 0$ and $0° < \alpha < 90°$.

2. Express $3\cos\theta + 2\sin\theta$ in the form $r\cos(\theta - \alpha)$, where $r > 0$ and $0° < \alpha < 90°$.

3. Express $\cos\theta + \sin\theta$ in the form $r\sin(\theta + \alpha)$, where $r > 0$ and $0 < \alpha < \frac{\pi}{2}$.

4. Express $\sin\theta - \sqrt{3}\cos\theta$ in the form $r\sin(\theta - \alpha)$, where $r > 0$ and $0 < \alpha < \frac{\pi}{2}$.

5. Express $\sqrt{3}\sin\theta + \cos\theta$ in the form $r\sin(\theta + \alpha)$, where $r > 0$ and $0 < \alpha < \frac{\pi}{2}$.
   Hence solve $\sqrt{3}\sin\theta + \cos\theta = 2$ for $0 \le \theta \le 2\pi$.
   Describe the transformations that need to be performed on the graph of $y = \sin\theta$ in order to obtain the graph of $y = \sqrt{3}\sin\theta + \cos\theta$.

6. Express $5\sin\theta - 12\cos\theta$ in the form $r\sin(\theta - \alpha)$, where $r > 0$ and $0° < \alpha < 90°$.
   Hence solve $5\sin\theta - 12\cos\theta = 3$ for $0° \le \theta \le 360°$.
   Describe the transformations that need to be performed on the graph of $y = \sin\theta$ in order to obtain the graph of $y = 5\sin\theta - 12\cos\theta$.

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Solutions to section test

1. $2\cos\theta - 3\sin\theta = r\cos(\theta+\alpha)$

$= r\cos\theta\cos\alpha - r\sin\theta\sin\alpha$

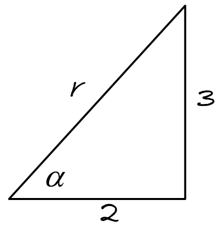
$r\cos\alpha = 2$

$r\sin\alpha = 3$

$r^2 = 2^2 + 3^2 = 13 \Rightarrow r = \sqrt{13}$

$\tan\alpha = \frac{3}{2} \Rightarrow \alpha = 56.3°$

$2\cos\theta - 3\sin\theta = \sqrt{13}\cos(\theta + 56.3°)$



2. $3\cos\theta + 2\sin\theta = r\cos(\theta-\alpha)$

$= r\cos\theta\cos\alpha + r\sin\theta\sin\alpha$

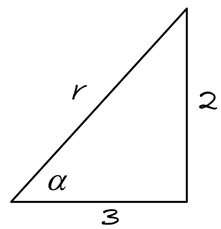
$r\cos\alpha = 3$

$r\sin\alpha = 2$

$r^2 = 3^2 + 2^2 = 13 \Rightarrow r = \sqrt{13}$

$\tan\alpha = \frac{2}{3} \Rightarrow \alpha = 33.7°$

$3\cos\theta + 2\sin\theta = \sqrt{13}\cos(\theta - 33.7°)$



3. $\cos\theta + \sin\theta = r\sin(\theta+\alpha)$

$= r\sin\theta\cos\alpha + r\cos\theta\sin\alpha$

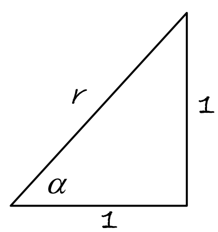
$r\cos\alpha = 1$

$r\sin\alpha = 1$

$r^2 = 1^2 + 1^2 = 2 \Rightarrow r = \sqrt{2}$

$\tan\alpha = 1 \Rightarrow \alpha = \frac{\pi}{4}$

$\cos\theta + \sin\theta = \sqrt{2}\sin\left(\theta + \frac{\pi}{4}\right)$



4. $\sin\theta - \sqrt{3}\cos\theta = r\sin(\theta-\alpha)$

$= r\sin\theta\cos\alpha - r\cos\theta\sin\alpha$

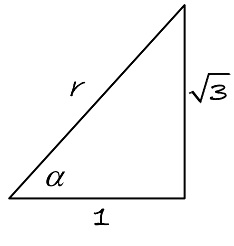
$r\cos\alpha = 1$

$r\sin\alpha = \sqrt{3}$

$r^2 = 1^2 + \sqrt{3}^2 = 4 \Rightarrow r = 2$

$\tan\alpha = \sqrt{3} \Rightarrow \alpha = \frac{\pi}{3}$

$\sin\theta - \sqrt{3}\cos\theta = 2\sin\left(\theta - \frac{\pi}{3}\right)$



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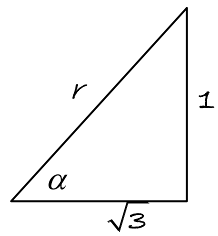
5. $\sqrt{3}\sin\theta + \cos\theta = r\sin(\theta+\alpha)$

$= r\sin\theta\cos\alpha + r\cos\theta\sin\alpha$



$r\cos\alpha = \sqrt{3}$

$r\sin\alpha = 1$

$r^2 = \sqrt{3}^2 + 1^2 = 4 \Rightarrow r = 2$

$\tan\alpha = \frac{1}{\sqrt{3}} \Rightarrow \alpha = \frac{\pi}{6}$

$\sqrt{3}\sin\theta + \cos\theta = 2\sin\left(\theta + \frac{\pi}{6}\right)$

$\sqrt{3}\sin\theta + \cos\theta = 2$

$2\sin\left(\theta + \frac{\pi}{6}\right) = 2$

$\sin\left(\theta + \frac{\pi}{6}\right) = 1$

$\theta + \frac{\pi}{6} = \frac{\pi}{2}$

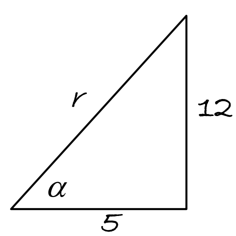
$\theta = \frac{\pi}{3}$

$\sqrt{3}\sin\theta + \cos\theta = 2\sin\left(\theta + \frac{\pi}{6}\right)$

To transform the graph of $y = \sin\theta$ to the graph of $y = 2\sin\left(\theta + \frac{\pi}{6}\right)$:
the factor of 2 represents a stretch scale factor 2 parallel to the y axis
and replacing θ by $\theta + \frac{\pi}{6}$ represents a horizontal translation of $\frac{\pi}{6}$ units to the left.

6. $5\sin\theta - 12\cos\theta = r\sin(\theta-\alpha)$

$= r\sin\theta\cos\alpha - r\cos\theta\sin\alpha$



$r\cos\alpha = 5$

$r\sin\alpha = 12$

$r^2 = 5^2 + 12^2 = 169 \Rightarrow r = 13$

$\tan\alpha = \frac{12}{5} \Rightarrow \alpha = 67.38°$

$5\sin\theta - 12\cos\theta = 13\sin(\theta - 67.4°)$

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$5\sin\theta - 12\cos\theta = 3$

$13\sin(\theta - 67.38°) = 3$

$\sin(\theta - 67.38°) = \frac{3}{13}$

$\theta - 67.38° = 13.34°$ or $166.66°$

$\theta = 80.7°$ or $234.0°$

To transform the graph of $y = \sin\theta$ to the graph of $y = 13\sin(\theta - 67.4°)$:
the factor of 13 represents a stretch scale factor 13 parallel to the y axis
and replacing θ by $\theta - 67.4°$ represents a horizontal translation of 67.4° units to the right.