

Section 1: The compound angle identities

Section test

1. The expansion of $\cos(60^\circ - \theta)$ is:

(a) $\frac{\sqrt{3}}{2} \cos \theta - \frac{1}{2} \sin \theta$

(b) $\frac{\sqrt{3}}{2} \cos \theta + \frac{1}{2} \sin \theta$

(c) $\frac{1}{2} \cos \theta - \frac{\sqrt{3}}{2} \sin \theta$

(d) $\frac{1}{2} \cos \theta + \frac{\sqrt{3}}{2} \sin \theta$

2. The expansion of $\sin(60^\circ - \theta)$ is:

(a) $\frac{\sqrt{3}}{2} \cos \theta + \frac{1}{2} \sin \theta$

(b) $\frac{\sqrt{3}}{2} \cos \theta - \frac{1}{2} \sin \theta$

(c) $\frac{1}{2} \cos \theta + \frac{\sqrt{3}}{2} \sin \theta$

(d) $\frac{1}{2} \cos \theta - \frac{\sqrt{3}}{2} \sin \theta$

3. The expansion of $\tan(60^\circ - \theta)$ is:

4.

(a) $\frac{\sqrt{3} + \tan \theta}{1 - \sqrt{3} \tan \theta}$

(b) $\frac{\sqrt{3} - \tan \theta}{1 + \sqrt{3} \tan \theta}$

(c) $\frac{1 + \sqrt{3} \tan \theta}{\sqrt{3} - \tan \theta}$

(d) $\frac{1 - \sqrt{3} \tan \theta}{\sqrt{3} + \tan \theta}$

5. Write the expression $\cos 3\theta \cos \theta - \sin 3\theta \sin \theta$ as a single sine or cosine.

6. Write the expression $\cos 4\theta \sin \theta - \sin 4\theta \cos \theta$ as a single sine or cosine.

7. Solve the equation $\sin(60^\circ + \theta) = \cos \theta$ for $0^\circ \leq \theta \leq 360^\circ$.

8. Solve the equation $\tan(\theta + 45^\circ) = 6 \tan \theta$ for $0^\circ \leq \theta \leq 360^\circ$.

9. Solve the equation $\tan 2\theta = 3 \tan \theta$ for $0 < \theta < \pi$. Give your answers in terms of π .

10. Solve the equation $\sin 2\theta = \cos \theta$ for $-\pi \leq \theta \leq \pi$. Give your answers in terms of π .

11. Solve the equation $\cos 2\theta = \sin \theta$ for $0 \leq \theta \leq 2\pi$. Give your answers in terms of π .

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Solutions to section test

$$\begin{aligned} 1. \quad \cos(A-B) &= \cos A \cos B + \sin A \sin B \\ \cos(60^\circ - \theta) &= \cos 60^\circ \cos \theta + \sin 60^\circ \sin \theta \\ &= \frac{1}{2} \cos \theta + \frac{\sqrt{3}}{2} \sin \theta \end{aligned}$$

$$\begin{aligned} 2. \quad \sin(A-B) &= \sin A \cos B - \cos A \sin B \\ \sin(60^\circ - \theta) &= \sin 60^\circ \cos \theta - \cos 60^\circ \sin \theta \\ &= \frac{\sqrt{3}}{2} \cos \theta - \frac{1}{2} \sin \theta \end{aligned}$$

$$\begin{aligned} 3. \quad \tan(A-B) &= \frac{\tan A - \tan B}{1 + \tan A \tan B} \\ \tan(60^\circ - \theta) &= \frac{\tan 60^\circ - \tan \theta}{1 + \tan 60^\circ \tan \theta} \\ &= \frac{\sqrt{3} - \tan \theta}{1 + \sqrt{3} \tan \theta} \end{aligned}$$

$$\begin{aligned} 4. \quad \cos(A+B) &= \cos A \cos B - \sin A \sin B \\ \cos 3\theta \cos \theta - \sin 3\theta \sin \theta &= \cos(3\theta + \theta) \\ &= \cos 4\theta \end{aligned}$$

$$\begin{aligned} 5. \quad \sin(A-B) &= \sin A \cos B - \cos A \sin B \\ \cos 4\theta \sin \theta - \sin 4\theta \cos \theta &= \sin(\theta - 4\theta) \\ &= \sin(-3\theta) \\ &= -\sin 3\theta \end{aligned}$$

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6. $\sin(60^\circ + \theta) = \cos \theta$

$$\sin 60^\circ \cos \theta + \cos 60^\circ \sin \theta = \cos \theta$$

$$\frac{\sqrt{3}}{2} \cos \theta + \frac{1}{2} \sin \theta = \cos \theta$$

$$\sqrt{3} \cos \theta + \sin \theta = 2 \cos \theta$$

$$\sin \theta = (2 - \sqrt{3}) \cos \theta$$

$$\tan \theta = 2 - \sqrt{3}$$

$$\theta = 15^\circ \text{ or } 195^\circ$$

7. $\tan(\theta + 45^\circ) = 6 \tan \theta$

$$\frac{\tan \theta + \tan 45^\circ}{1 - \tan \theta \tan 45^\circ} = 6 \tan \theta$$

$$\tan \theta + 1 = 6 \tan \theta (1 - \tan \theta)$$

$$\tan \theta + 1 = 6 \tan \theta - 6 \tan^2 \theta$$

$$6 \tan^2 \theta - 5 \tan \theta + 1 = 0$$

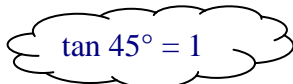
$$(3 \tan \theta - 1)(2 \tan \theta - 1) = 0$$

$$\tan \theta = \frac{1}{3} \text{ or } \tan \theta = \frac{1}{2}$$

$$\tan \theta = \frac{1}{3} \Rightarrow \theta = 18.4^\circ \text{ or } 198.4^\circ$$

$$\tan \theta = \frac{1}{2} \Rightarrow \theta = 26.6^\circ \text{ or } 206.6^\circ$$

The roots are $18.4^\circ, 26.6^\circ, 198.4^\circ, 206.6^\circ$.



$\tan 45^\circ = 1$

8. $\tan 2\theta = 3 \tan \theta$

$$\frac{2 \tan \theta}{1 - \tan^2 \theta} = 3 \tan \theta$$

$$2 \tan \theta = 3 \tan \theta (1 - \tan^2 \theta)$$

$$2 \tan \theta = 3 \tan \theta - 3 \tan^3 \theta$$

$$3 \tan^3 \theta - \tan \theta = 0$$

$$\tan \theta (3 \tan^2 \theta - 1) = 0$$

$$\tan \theta = 0 \text{ or } \pm \frac{1}{\sqrt{3}}$$

$$\tan \theta = 0 \Rightarrow \theta = 0, \pi \text{ (not in required range)}$$

$$\tan \theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = \frac{\pi}{6}$$

$$\tan \theta = -\frac{1}{\sqrt{3}} \Rightarrow \theta = \frac{5\pi}{6}$$

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The roots are $\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

9. $\sin 2\theta = \cos \theta$

$$2\sin \theta \cos \theta = \cos \theta$$

$$2\sin \theta \cos \theta - \cos \theta = 0$$

$$\cos \theta (2\sin \theta - 1) = 0$$

$$\cos \theta = 0 \text{ or } \sin \theta = \frac{1}{2}$$

$$\cos \theta = 0 \Rightarrow \theta = -\frac{\pi}{2} \text{ or } \frac{\pi}{2}$$

$$\sin \theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6} \text{ or } \frac{5\pi}{6}$$

The roots are $-\frac{\pi}{2}, \frac{\pi}{6}, \frac{\pi}{2}$ or $\frac{5\pi}{6}$

10. $\cos 2\theta = \sin \theta$

$$1 - 2\sin^2 \theta = \sin \theta$$

$$2\sin^2 \theta + \sin \theta - 1 = 0$$

$$(2\sin \theta - 1)(\sin \theta + 1) = 0$$

$$\sin \theta = \frac{1}{2} \text{ or } -1$$

$$\sin \theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$\sin \theta = -1 \Rightarrow \theta = \frac{3\pi}{2}$$

The roots are $\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$.