## Edexcel A level Mathematics Trigonometry

## Section 2: Circular measure

## Section test

1. Find the perimeter and area of this sector.

2. A sector of a circle with radius 5 cm has arc length 12 cm . Find the angle of the sector, in radians.
3. A sector of a circle has perimeter 28 cm and area $48 \mathrm{~cm}^{2}$. Find two possible values for the radius of the circle.
4. Find the area of the shaded segment.

5. The arc length of a sector of radius 8 cm is $3 \pi \mathrm{~cm}$.

Find the area of the sector, giving your answers in terms of $\pi$.
6. The area of a sector with an angle of $45^{\circ}$ is $2 \pi \mathrm{~cm}^{2}$.

Find its perimeter. Give your answer correct to 2 d.p.
7. The expression $\sin 3 x+2 \cos x$ can be approximated for small values of $x$ by the polynomial $a+b x+c x^{2}$. Find the values of $a, b$ and $c$.
8. The expression $x \tan x+\cos 2 x-3 \sin 4 x$ can be approximated for small values of $x$ by the polynomial $a+b x+c x^{2}$. Find the values of $a, b$ and $c$.
9. Use small angle approximations to find an approximation to the smallest positive root of the equation $\cos x=6 x$.

## Solutions to section test

1. $30^{\circ}=30 \times \frac{\pi}{180}=\frac{\pi}{6}$ radians

Arc length $=r \theta=5 \times \frac{\pi}{6}=2.62 \mathrm{~cm}$
so perimeter $=2.62+5+5=12.6 \mathrm{~cm}$ (3s.f.)
Area $=\frac{1}{2} r^{2} \theta=\frac{1}{2} \times 5^{2} \times \frac{\pi}{6}=6.54 \mathrm{~cm}^{2}$ (3 s.f.)
2. Arc length $=r \theta$
$12=5 \theta$
$\theta=2.4$ radians
3. $2 r+r \theta=28 \Rightarrow r \theta=28-2 r$
$\frac{1}{2} r^{2} \theta=48$
$r^{2} \theta=96$
$r(28-2 r)=96$
$28 r-2 r^{2}=96$
$r^{2}-14 r+48=0$
$(r-6)(r-8)=0$
The radius is 6 cm or 8 cm .
4. Area of sector $=\frac{1}{2} r^{2} \theta=\frac{1}{2} \times 6^{2} \times \frac{\pi}{3}=6 \pi$

Triangle is equilateral since all angles are $60^{\circ}$
Height of triangle $=\sqrt{6^{2}-3^{3}}=\sqrt{27}$
Area of triangle $=3 \sqrt{27}$
Area of segement $=6 \pi-3 \sqrt{27}=3.26 \mathrm{~cm}^{2}$

5. Arc length $=r \theta \quad \Rightarrow 3 \pi=8 \theta$

$$
\Rightarrow \theta=\frac{3}{8} \pi
$$

Area of sector $=\frac{1}{2} r^{2} \theta=\frac{1}{2} \times 8^{2} \times \frac{3}{8} \pi=12 \pi \mathrm{~cm}^{2}$
6. $45^{\circ}=45 \times \frac{\pi}{180}=\frac{1}{4} \pi$

$$
\begin{array}{ll}
\text { Area }=\frac{1}{2} r^{2} \theta & \Rightarrow 2 \pi=\frac{1}{2} r^{2} \times \frac{1}{4} \pi \\
& \Rightarrow r^{2}=16 \\
& \Rightarrow r=4
\end{array}
$$

Arc length $=r \theta=4 \times \frac{1}{4} \pi=\pi$
Perimeter of sector $=\pi+4+4=(\pi+8) \mathrm{cm}$.
7. $\sin 3 x+2 \cos x \approx 3 x+2\left(1-\frac{1}{2} x^{2}\right)$

$$
=2+3 x-x^{2}
$$

$a=2, b=3, c=-1$
8. $x \tan x+\cos 2 x-3 \sin 4 x \approx x \times x+\left(1-\frac{1}{2}(2 x)^{2}-3 \times 4 x\right.$

$$
\begin{aligned}
& =x^{2}+1-2 x^{2}-12 x \\
& =1-12 x-x^{2}
\end{aligned}
$$

$a=1, b=-12, c=-1$
9. $\cos x=6 x$
using small angle approximations,

$$
\begin{aligned}
& 1-\frac{1}{2} x^{2}=6 x \\
& x^{2}+12 x-2=0 \\
& x=\frac{-12 \pm \sqrt{144-4 \times 1 \times-2}}{2}=\frac{-12 \pm \sqrt{152}}{2}
\end{aligned}
$$

smallest positive root $\approx 0.164$

