Section 1: Working with radians

Notes and Examples

In this section you learn about radians and circular measure.

These notes contain subsections on:
- Radians
- Common values of trig functions in radians
- Trigonometric graphs in radians
- Solving trigonometric equations using radians

Radians

Often mathematicians use radians rather than degrees to measure angles as they sometimes make calculations easier.

To convert degrees to radians you multiply by $\dfrac{\pi}{180°}$

To convert radians to degrees you multiply by $\dfrac{180°}{\pi}$

In the example below you need to convert degrees into radians.

Example 1
Convert these angles to radians:
(i) 60° (ii) 270° (iii) 173°

Solution

(i) $60° = 60° \times \dfrac{\pi}{180°} = \dfrac{\pi}{3}$

(ii) $270° = 270° \times \dfrac{\pi}{180°} = \dfrac{3\pi}{2}$

(iii) $173° = 173° \times \dfrac{\pi}{180°} = 3.02$ rads

In the next example you need to convert radians into degrees.

Example 2
Convert these angles to degrees:
(i) $\dfrac{\pi}{4}$ (ii) $\dfrac{7\pi}{10}$ (iii) 0.65 rads

Solution

(i) $\frac{\pi}{4} = \frac{\pi}{4} \times \frac{180°}{\pi} = 45°$

(ii) $\frac{7\pi}{10} = \frac{7\pi}{10} \times \frac{180°}{\pi} = 126°$

(iii) $0.65 \text{ rads} = 0.65 \times \frac{180°}{\pi} = 37.2°$

Common values of trig functions in radians

You should soon be able to remember the radian equivalent of many common angles:

$30° = \frac{\pi}{6}$ radians $\quad 45° = \frac{\pi}{4}$ radians $\quad 60° = \frac{\pi}{3}$ radians

$90° = \frac{\pi}{2}$ radians $\quad 180° = \pi$ radians $\quad 360° = 2\pi$ radians.

You should normally express these as fractions of π rather than as decimals.

Just as when working in degrees, you should be able to recognise the values of sin θ, cos θ and tan θ for common values of θ.

This table reminds you of the common values you should know, with the angles given in radians.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
tan θ	0	$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	undefined

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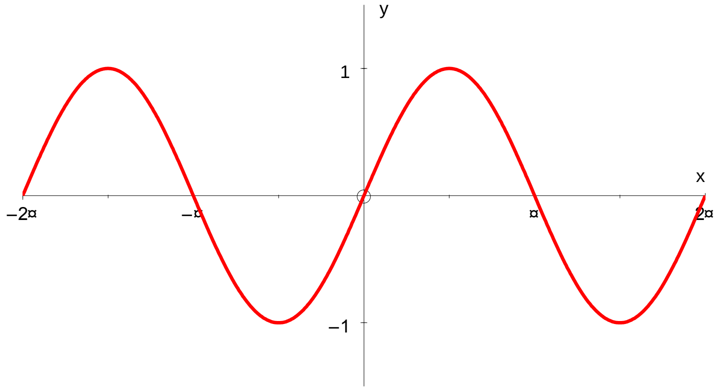
Trigonometric graphs in radians

You need to be familiar with the graphs of the trig functions when working in radians as well as in degrees. Here are reminders of these graphs.

The graph of y = sin θ where θ is in radians:

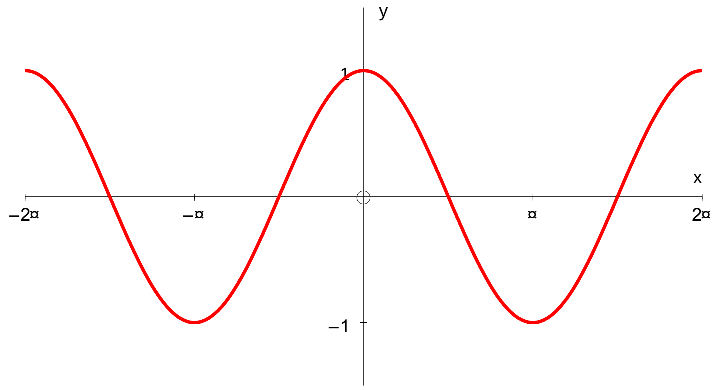


It repeats every 2π radians

$\frac{\pi}{6}\text{rad} = \frac{180^\circ}{6} = 30^\circ$

- The graph of $y = \sin x$ has a period of 2π.
  So for example, $\sin 0.2 = \sin(0.2 + 2\pi)$
- It has rotational symmetry about the origin.
  So for example, since $\sin\frac{\pi}{6} = 0.5$ then $\sin\left(-\frac{\pi}{6}\right) = -0.5$
- There is a line of symmetry at $\theta = \frac{\pi}{2}$ and $\theta = -\frac{\pi}{2}$

The graph of $y = \cos\theta$ where θ is in radians



- The graph of $y = \cos\theta$ has a period of 2π.
  So for example, $\cos 0.2 = \cos(0.2 + 2\pi)$
- It has line symmetry about the y-axis.
  So for example, $\cos\frac{\pi}{3} = \cos\left(-\frac{\pi}{3}\right)$

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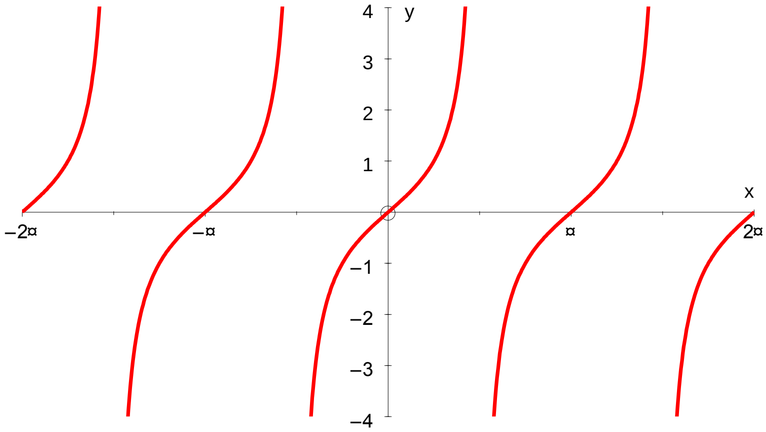
The graph of $y = \tan\theta$ where θ is in radians



- The graph of $y = \tan x$ has a period of π.
  So for example, $\tan 0.2 = \tan(0.2 + 2\pi)$
- It has rotational symmetry about the origin.
  So for example, since $\tan\frac{\pi}{3} = \sqrt{3}$ then $\tan\left(-\frac{\pi}{3}\right) = -\sqrt{3}$

$\frac{\pi}{3}\text{rad} = \frac{180^\circ}{3} = 60^\circ$

Solving trigonometric equations using radians

You have previously solved trigonometric equations, giving answers in degrees. The same techniques can be used when working in radians.
Make sure that your calculator is set to **Radians**. However, remember that you should be able to recognise common values of the trig functions and give the exact angle in radians as a fraction of π where appropriate.
In this example you need to find a solution to a trigonometric equation in radians.

Example 3
Solve $\cos\theta = 0.5$ for $0 \le \theta \le 2\pi$

When the range is given in radians then you must find a solution in radians.

Solution
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You know that cos 60°= 0.5

$$60^\circ = 60^\circ \times \frac{\pi}{180^\circ} = \frac{\pi}{3} \text{ radians}$$

So $\theta = \frac{\pi}{3}$ is one solution.

There is another solution in the 4th quadrant.

$2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$ is also a solution.

So the values of θ for which $\cos\theta = 0.5$ are $\frac{\pi}{3}$ and $\frac{5\pi}{3}$

In this example you need to use a trigonometric identity.

Example 4

Solve $2\sin\theta = \cos\theta$ for $-\pi \le \theta \le \pi$

Solution

You can divide by $\cos\theta$ since we know that $\cos\theta \ne 0$ as $\sin\theta$ and $\cos\theta$ are not both equal to 0 for the same value of θ.

So: $$\frac{2\sin\theta}{\cos\theta} = \frac{\cos\theta}{\cos\theta} \Rightarrow \frac{2\sin\theta}{\cos\theta} = 1 \qquad ①$$

You know $\tan\theta \equiv \dfrac{\sin\theta}{\cos\theta}$ ②

Substituting ② into ①: $$2\tan\theta \equiv 1$$
$$\Rightarrow \tan\theta = \frac{1}{2}$$
$$\Rightarrow \theta = 0.464 \text{ rads}$$

There is also a solution in the 3rd quadrant.

0.464 - π = -2.68 rads is also a solution.

So the values of θ for which $2\sin\theta = \cos\theta$ are -2.68 rads and 0.464 rads.

In this example you need to use the identity $\sin^2\theta + \cos^2\theta \equiv 1$.

Example 5

Solve $3 + 3\sin\theta = 2\cos^2\theta$ for $0 \le \theta \le 2\pi$

Solution

Using the identity $\sin^2\theta + \cos^2\theta \equiv 1$ gives: $\cos^2\theta \equiv 1 - \sin^2\theta$ ①

Substituting ① into $3 + 3\sin\theta = 2\cos^2\theta$: $3 + 3\sin\theta = 2(1 - \sin^2\theta)$

$3 + 3\sin\theta = 2 - 2\sin^2\theta$

This is a quadratic in $\sin\theta$ which rearranges to: $2\sin^2\theta + 3\sin\theta + 1 = 0$

Factorising: $(2\sin\theta + 1)(\sin\theta + 1) = 0$

So either: $$(2\sin\theta + 1) = 0 \Rightarrow \sin\theta = -\frac{1}{2}$$
$$\Rightarrow \theta = -\frac{\pi}{6} \text{ or } 2\pi + \left(-\frac{\pi}{6}\right) = \frac{11\pi}{6}$$
$$\text{or } \theta = \pi - \left(-\frac{\pi}{6}\right) = \frac{7\pi}{6}$$

or $$(\sin\theta + 1) = 0 \quad \Rightarrow \sin\theta = -1$$
$$\Rightarrow \theta = -\frac{\pi}{2} \ or\ -\frac{\pi}{2} + 2\pi = \frac{3\pi}{2}$$

So the solutions to $3 + 3\sin\theta = 2\cos^2\theta$ are $\theta = \dfrac{7\pi}{6}, \dfrac{11\pi}{6}$ or $\dfrac{3\pi}{2}$