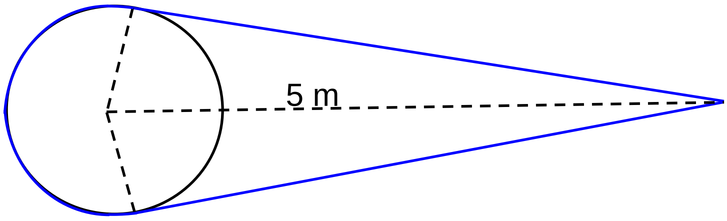
Topic assessment

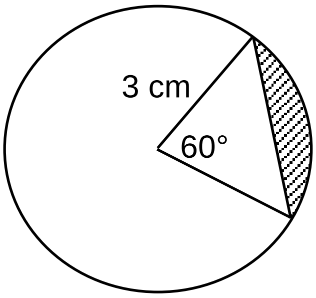
1. A belt is wrapped around a cylinder of radius 2.5 m as shown.



Find the length of the belt. [6]

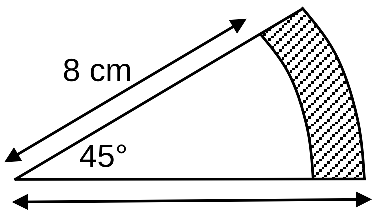
2. Find the perimeter and area of the shaded sections of these shapes.

(i)



[7]

(ii)



[7]

3. (i) Sketch the graph of $y = \cos x$ for $-\pi \le x \le \pi$, [2]
(ii) Sketch the line $y = 3x$ on the same axes, and indicate the point where the graphs intersect. [1]
(iii) Use small angle approximations to find an approximate value for the x-coordinate of the intersection point. [5]

4. Solve these equations for $0 \le \theta \le 2\pi$.
Give your answers as a multiple of π .
(i) $\cos\theta = \dfrac{\sqrt{3}}{2}$ [2]
(ii) $\sin\theta = 0.5$ [2]
(iii) $\tan\theta = \sqrt{3}$ [2]

5. Solve these equations for $0 \le \theta \le 2\pi$.
Give your answers as a multiple of π.

(i) $\cos^2\theta = \dfrac{3}{4}$ [3]

(ii) $3\tan^2\theta = 1$ [3]

Total 40 marks

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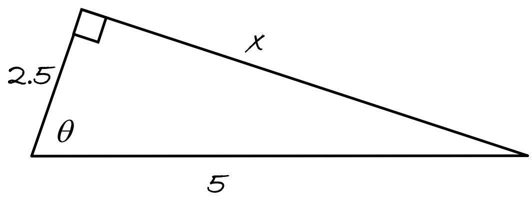
Topic Assessment solutions

1. $\cos\theta = \dfrac{\text{Adjacent}}{\text{Hypotenuse}} = \dfrac{2.5}{5} = \dfrac{1}{2}$

$\theta = 60°$



Angle of arc with belt on $= 360° - 60° - 60° = 240°$

$240° = 240 \times \dfrac{\pi}{180} = \dfrac{4\pi}{3}$

Arc length $= r\theta = 2.5\left(\dfrac{4\pi}{3}\right) = \dfrac{10\pi}{3}$

$\tan\theta = \dfrac{x}{2.5}$

$x = 2.5\tan 60°$

Total length of belt $= \dfrac{10\pi}{3} + 2x$

$= \dfrac{10\pi}{3} + 5\tan 60°$

$= 19.1$ m (3 s.f.)

[6]

2. (i) If the triangle has an angle of $60°$ at the centre of the circle then it must be an equilateral triangle and so part of the perimeter is 3cm.

$60° = 60 \times \dfrac{\pi}{180} = \dfrac{\pi}{3}$

Arc length $= r\theta = 3\left(\dfrac{\pi}{3}\right) = \pi$

Perimeter $= (\pi + 3)$ cm

$= 6.14$ cm (3 s.f.)

Area of sector $= \frac{1}{2}r^2\theta = 0.5 \times 3^2 \times \dfrac{\pi}{3} = \dfrac{3\pi}{2}$

Area $=$ Area of sector $-$ Area of triangle

$= \dfrac{3\pi}{2} - \dfrac{1}{2} \times 3 \times 3\sin 60°$

$= 0.815$ cm^2 (3 s.f.)

[7]

(ii) Arc length $= r\theta$

Sector area $= \frac{1}{2}r^2\theta$

$45° = 45 \times \dfrac{\pi}{180} = \dfrac{\pi}{4}$

Sector 1: $\quad$ Arc length $= 10 \times \frac{\pi}{4} = \frac{5\pi}{2}$

Sector area $= \frac{1}{2} \times 10^2 \times \frac{\pi}{4} = \frac{25\pi}{2}$

Sector 2: $\quad$ Arc length $= 8 \times \frac{\pi}{4} = 2\pi$

Sector area $= \frac{1}{2} \times 8^2 \times \frac{\pi}{4} = 8\pi$

Shaded area: $\quad$ Perimeter $= 2 + 2 +$ Arc length 1 $+$ Arc length 2

$$= 4 + \frac{5\pi}{2} + 2\pi$$

$$= 18.1 \text{ cm (3 s.f.)}$$

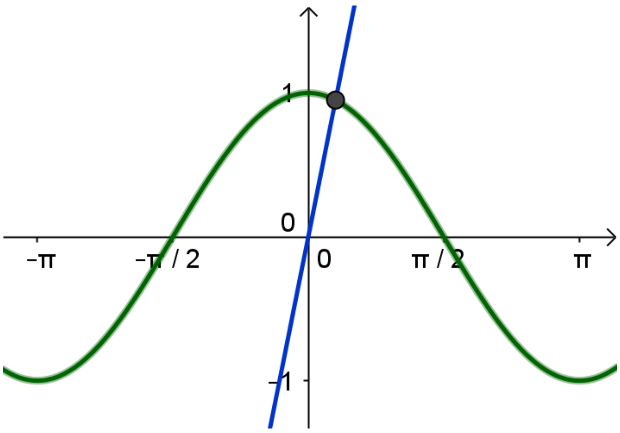
Area $=$ Area of sector 1 $-$ Area of sector 2

$$= \frac{25\pi}{2} - 8\pi$$

$$= 14.1 \text{ cm}^2 \text{(3 s.f.)}$$

[7]

3. (i), (ii)



(iii) $\cos x = 3x$

$1 - \frac{1}{2}x^2 \approx 3x$

$x^2 + 6x - 2 = 0$

$x = \frac{-6 \pm \sqrt{36 - 4 \times 1 \times -2}}{2} = \frac{-6 \pm \sqrt{44}}{2} = -3 \pm \sqrt{11}$

The roots of the quadratic equation are 0.317 and -6.32

The root -6.32 arises because the cosine graph is being approximated by a quadratic, and the line would cut this quadratic twice.

The required root is the positive on, and it is approximately 0.317.

4. (i) $\cos\theta = \frac{\sqrt{3}}{2}$

Solutions are in 1st and 4th quadrants.

$\theta = \frac{\pi}{6}$ or $\theta = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$

$\theta = \frac{\pi}{6}, \frac{11\pi}{6}$

[2]

(ii) $\sin\theta = 0.5$

Solutions are in 1st and 2nd quadrants

$\theta = \frac{\pi}{6}$ or $\theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

[2]

(iii) $\tan\theta = \sqrt{3}$

Solutions are in 1st and 3rd quadrants

$\theta = \frac{\pi}{3}$ or $\theta = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$

$\theta = \frac{\pi}{3}, \frac{4\pi}{3}$

[2]

5. (i) $\cos^2\theta = \frac{3}{4}$

$\cos\theta = \pm\frac{\sqrt{3}}{2}$

$\cos\theta = \frac{\sqrt{3}}{2}$ has solutions in the 1st and 4th quadrants

$\theta = \frac{\pi}{6}$ or $\theta = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$

$\cos\theta = -\frac{\sqrt{3}}{2}$ has solutions in the 2nd and 4th quadrants

$\theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$ or $\theta = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

[3]

(ii) $3\tan^2\theta = 1$

$\tan^2\theta = \frac{1}{3}$

$\tan\theta = \pm\frac{1}{\sqrt{3}}$

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$\tan\theta = \frac{1}{\sqrt{3}}$ has solutions in the 1st and 3rd quadrants

$$\theta = \frac{\pi}{6} \text{ or } \theta = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$$

$\tan\theta = -\frac{1}{\sqrt{3}}$ has solutions in the 2nd and 4th quadrants

$$\theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6} \text{ or } \theta = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$$

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

[3]