Topic assessment

1. An arithmetic series has first term 3 and common difference 5. Find
   (i) the 4th term [2]
   (ii) the sum of the first 12 terms. [3]

2. The 5th term of an arithmetic series is 16 and the 10th term is 30.
   (i) Find the first term and the common difference. [4]
   (ii) How many terms of the series are needed for the sum of the series to exceed 1000? [4]

3. A geometric series has first term 2 and common ratio 0.2. Find
   (i) the 3rd term [2]
   (ii) the sum of the first 4 terms of the series [3]
   (iii) the sum to infinity of the series. [2]

4. A geometric series has 1st term 3 and sum to infinity 8.
   Find the common ratio. [4]

5. A geometric series has first term 54 and 4th term 2.
   (i) What is the common ratio? [3]
   (ii) Find the sum to infinity of the series. [2]
   (iii) After how many terms is the sum of the series greater than 99% of the sum to infinity? [5]

6. When Mirka is 5 years old, her parents start to give her pocket money of 50p per week. On her birthday each year, her parents increase her pocket money by 50p.
   (i) How much pocket money does Mirka get in the first year? [2]
   (ii) How much more money in total does Mirka get in the second year than the first year? [3]
   (iii) How much money has Mirka been given in total by her 11th birthday? [3]
   (iv) After how many complete years is the total amount Mirka has been given more than £1000? [4]

7. At the beginning of each month, Mark puts £N from his salary into a savings account. At the end of every month, interest is added to his savings at the rate of r% per month.
   (i) Write down an expression for the amount of money in Mark's account at the end of (a) 1 month (b) 2 months (c) 3 months,
   and hence show that the amount of money in Mark's account at the end of n months is given by
   $$N\left(1+\tfrac{r}{100}\right)+N\left(1+\tfrac{r}{100}\right)^2+N\left(1+\tfrac{r}{100}\right)^3+....+N\left(1+\tfrac{r}{100}\right)^n$$ [8]
   (ii) Use the formula for a geometric progression to simplify this expression. [3]
   (iii) How much does Mark have after 5 years if he saves £100 a month at an interest rate of 0.5% per month? [3]

Total 60 marks

Edexcel A level Maths Sequences Assessment solns

Solutions to topic assessment

1. $a = 3,\ d = 5$

(i) $a_k = a + (k-1)d$

$a_4 = 3 + 3(5)$

$= 3 + 15$

$= 18$

[2]

(ii) $S_n = \frac{1}{2}n[2a + (n-1)d]$

$S_{12} = \frac{1}{2} \times 12[(2 \times 3) + (11 \times 5)]$

$= 6[6 + 55]$

$= 6 \times 61$

$= 366$

[3]

2. $a_5 = 16,\ a_{10} = 30$

(i) $16 = a + 4d$ ①

$30 = a + 9d$ ②

② − ①: $14 = 5d$

$d = 2.8$

$16 = a + 4(2.8)$

$16 = a + 11.2$

$a = 4.8$

[4]

(ii) $S_n = \frac{1}{2}n[2a + (n-1)d]$

$1000 = \frac{1}{2}n[2(4.8) + (n-1)(2.8)]$

$1000 = \frac{1}{2}n[9.6 + 2.8n - 2.8]$

$1000 = \frac{1}{2}n[6.8 + 2.8n]$

$2000 = n[6.8 + 2.8n]$

$2000 = 6.8n + 2.8n^2$

$2.8n^2 + 6.8n - 2000 = 0$

$$n = \frac{-6.8 \pm \sqrt{6.8^2 - (4 \times 2.8 \times -2000)}}{2 \times 2.8}$$

$$n = \frac{-6.8 \pm \sqrt{22446.24}}{5.6}$$

$n = 25.53$ (n cannot be negative)

The number of terms needed is therefore be 26.

Check: $S_{25} = \frac{1}{2} \times 25\left[(2 \times 4.8) + (25-1)(2.8)\right]$

$= 960$

$S_{26} = \frac{1}{2} \times 26\left[(2 \times 4.8) + (26-1)(2.8)\right]$

$= 1034.8$

[4]

3. $a = 2,\ r = 0.2$

(i) $a_k = ar^{k-1}$

$a_3 = (2)(0.2^2)$

$a_3 = 0.08$

[2]

(ii) $S_n = \dfrac{a(1-r^n)}{1-r}$

$S_4 = \dfrac{2(1-(0.2)^4)}{1-0.2}$

$S_4 = 2.496$

[3]

(iii) $S_\infty = \dfrac{a}{1-r}$

$S_\infty = \dfrac{2}{1-0.2}$

$S_\infty = 2.5$

[2]

4. $a = 3,\ S_\infty = 8$

$S_\infty = \dfrac{a}{1-r}$

$8 = \dfrac{3}{1-r}$

$8(1-r) = 3$

$1 - r = \frac{3}{8}$

$r = 1 - \frac{3}{8}$

$r = \frac{5}{8}$

[4]

5. $a = 54,\ a_4 = 2$

(i) $a_k = ar^{k-1}$

$2 = 54(r^3)$

$r^3 = \frac{1}{27}$

$r = \frac{1}{3}$

[3]

(ii) $S_\infty = \frac{a}{1-r}$

$S_\infty = \frac{54}{1-\frac{1}{3}}$

$S_\infty = 81$

[2]

(iii) $S_\infty = 81$

$\frac{99}{100} \times 81 = 80.19$

$S_n = \frac{a(1-r^n)}{1-r}$

$80.19 < \frac{54\left(1-\left(\frac{1}{3}\right)^n\right)}{1-\left(\frac{1}{3}\right)}$

$54\left(1-\left(\frac{1}{3}\right)^n\right) > 80.19\left(\frac{2}{3}\right)$

$1-\left(\frac{1}{3}\right)^n > \frac{53.46}{54}$

$\left(\frac{1}{3}\right)^n < 0.01$

There are 2 methods that you can use to solve this.

Method 1: Trial and error

$n = 10, \left(\frac{1}{3}\right)^{10} = 0.0000169$

$n = 5, \left(\frac{1}{3}\right)^{5} = 0.00411$

$n = 4, \left(\frac{1}{3}\right)^{4} = 0.0123$

Method 2: Using logarithms

$\left(\frac{1}{3}\right)^n < 0.01$

$\log\left(\frac{1}{3}\right)^n < \log 0.01$

$n\log\left(\frac{1}{3}\right) < \log 0.01$

$n > \frac{\log 0.01}{\log \frac{1}{3}}$

$n > 4.191...$

The logarithm of a number less than 1 is negative. Since you are dividing by a negative number, the inequality must be reversed.

The sum is more than 99% of the sum to infinity after the 5th term.

[5]

6. (i) There are 52 weeks in a year, and so the total amount of pocket money in the first year is: $52 \times £0.5 = £26$

[2]

(ii) In the second year, there are still 52 weeks of pocket money, but it has increased by 50p each week so the increase overall is: $52 \times £0.5 = £26$

[3]

(iii) $a = 26, \ d = 26$

$$S_n = \tfrac{1}{2}n[2a + (n-1)d]$$
$$S_6 = \tfrac{1}{2} \times 6[(2 \times 26) + (6-1)(26)]$$
$$= 3[52 + 130]$$
$$= 3 \times 182$$
$$= 546$$

Mirka has been given £546 in total by her 11th birthday.

[3]

(iv) $S_n = \tfrac{1}{2}n[2a + (n-1)d]$

$$1000 = \tfrac{1}{2}n[(2 \times 26) + (n-1)(26)]$$
$$2000 = n[52 + 26n - 26]$$
$$2000 = n[26n + 26]$$
$$2000 = 26n^2 + 26n$$
$$1000 = 13n^2 + 13$$
$$13n^2 + 13n - 1000 = 0$$
$$n = \frac{-13 \pm \sqrt{13^2 - (4 \times 13 \times -1000)}}{2 \times 13}$$
$$n = \frac{-13 \pm \sqrt{169 + 52000}}{26}$$

$n = -9.28$ or $n = 8.284$

n cannot be a negative number, and so in complete years, n must equal 9.

Check: $S_8 = \tfrac{1}{2} \times 8[(2 \times 26) + (7 \times 26)]$

$= 936$

$S_9 = \tfrac{1}{2} \times 9[(2 \times 26) + (8 \times 26)]$

$= 1170$

Mirka has been given more than £1000 after 9 complete years.

[4]

7. As r is a percentage, adding interest of r% is equivalent to multiplying the total amount by $1 + \frac{r}{100}$, so:

(i) (a) 1st month: Money $= N\left(1 + \frac{r}{100}\right)$

(b) 2nd month: Money $= \left(N + N\left(1 + \frac{r}{100}\right)\right) \times \left(1 + \frac{r}{100}\right)$

$$= N\left(1 + \frac{r}{100}\right) + N\left(1 + \frac{r}{100}\right)^2$$

Another N is added to last month's total, and the whole amount is multiplied by $1 + \frac{r}{100}$ for the interest.

(c) 3rd month: $\text{Money} = \left(N + N\left(1+\frac{r}{100}\right) + N\left(1+\frac{r}{100}\right)^2\right) \times \left(1+\frac{r}{100}\right)$

$$= N\left(1+\frac{r}{100}\right) + N\left(1+\frac{r}{100}\right)^2 + N\left(1+\frac{r}{100}\right)^3$$

> As before, add another N and multiply the whole amount by $1+\frac{r}{100}$.

Looking at these results, you can see that the general result for the total after n month is:

$$N\left(1+\frac{r}{100}\right) + N\left(1+\frac{r}{100}\right)^1 + N\left(1+\frac{r}{100}\right)^2 + N\left(1+\frac{r}{100}\right)^3 + \ldots + N\left(1+\frac{r}{100}\right)^n$$

[8]

(ii) This is a geometric progression with $a = N\left(1+\frac{r}{100}\right)$ and $r = \left(1+\frac{r}{100}\right)$

$$S_n = \frac{a\left(r^n - 1\right)}{r-1}$$

$$S_n = \frac{N\left(1+\frac{r}{100}\right)\left(\left(1+\frac{r}{100}\right)^n - 1\right)}{1+\frac{r}{100}-1}$$

$$S_n = \frac{100N\left(1+\frac{r}{100}\right)\left(\left(1+\frac{r}{100}\right)^n - 1\right)}{r}$$

[3]

(iii) After 5 years, a total of 60 months have passed so $n = 60$. If the interest rate is 0.5%, then $r = 0.5$. If Mark puts in £100 a month, then $N = 100$.

$$S_{60} = \frac{(100\times100)\left(1+\frac{0.5}{100}\right)\left(\left(1+\frac{0.5}{100}\right)^{60} - 1\right)}{0.5}$$

$$S_{60} = \frac{(10000)(1.005)\left((1.005)^{60} - 1\right)}{0.5}$$

$$S_{60} = 7011.888\ldots$$

$$S_{60} = £7011.89$$

[3]