Topic assessment

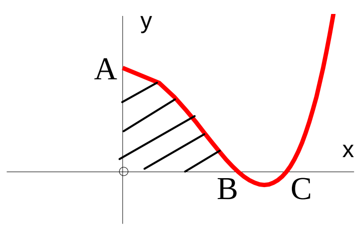
1. A curve is defined by the parametric equations $x = 2t^2,\ y = 4t$.
   (i) By eliminating the parameter, find the cartesian equation of the curve. [3]
   (ii) Find the equation of the tangent to the curve at the point A with parameter $t = 2$. [4]
   (iii) Show that the tangent does not meets the curve again. [3]
   (iv) The normal of the curve at A cuts the curve again at B. Find the coordinates of B. [5]

2. Find the turning points of the curve with parametric equations $x = 3t,\ y = 12t - t^3$ and distinguish between them. [6]

3. A circle is defined by the parametric equations $x = 1 + 2\cos\theta,\ y = 3 + 2\sin\theta$.
   (i) Sketch the circle. [2]
   (ii) Find $\frac{dy}{dx}$ at the point with parameter θ. [3]
   (iii) Find the equation of the tangent at the point with parameter θ. [3]
   (iv) Find the coordinates of the point where $\theta = \frac{\pi}{3}$. [2]
   (v) Find the equation of the normal at the point where $\theta = \frac{\pi}{3}$. [4]

4. A line is defined by the parametric equations $x = \cos 2t,\ y = \sin^2 t$
   (i) Find $\frac{dy}{dx}$. [3]
   (ii) Find the cartesian equation of the line. [3]

5. The diagram below shows the curve given by the parametric equations
   $x = 2\sqrt{t},\ y = t^2 - 3t + 2$.



   (i) Find the coordinates of the points A, B and C. [3]
   (ii) Find the shaded area. [6]

Total 50 marks

Edexcel A level Maths Parametric Assessment solutions

Solutions to topic assessment

1. (i) $x = 2t^2$ ①

$y = 4t \Rightarrow t = \frac{y}{4}$ ②

Insert ② into ①: $x = 2\left(\frac{y}{4}\right)^2 \Rightarrow x = \frac{y^2}{8}$

$\Rightarrow y^2 = 8x$

[3]

(ii) $x = 2t^2 \Rightarrow \frac{dx}{dt} = 4t$

$y = 4t \Rightarrow \frac{dy}{dt} = 4$

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{4}{4t} = \frac{1}{t}$

When $t = 2$: $\frac{dy}{dx} = \frac{1}{2}$

When t = 2: $x = 2 \times 2^2 = 8$ and $y = 4 \times 2 = 8$

So the tangent has gradient $\frac{1}{2}$ and passes through (8, 8)

Equation of tangent is $y - 8 = \frac{1}{2}(x - 8)$

$2y - 16 = x - 8$

$2y = x + 8$

[4]

(iii) Where tangent meets curve, $2 \times 4t = 2t^2 + 8$

$2t^2 - 8t + 8 = 0$

$t^2 - 4t + 4 = 0$

$(t - 2)^2 = 0$

The only root is the repeated root at $t = 2$ (the point of the tangent) so the tangent doesn't meet the curve again.

[3]

(iv) Gradient of tangent $= \frac{1}{2}$, so gradient of normal $= -2$ (using $m_1 m_2 = -1$)

Normal has gradient -2 and passes through (8, 8)

Equation of normal is $y - 8 = -2(x - 8)$

$y - 8 = -2x + 16$

$y + 2x = 24$

Where normal meets curve, $4t + 2 \times 2t^2 = 24$

$$4t + 4t^2 = 24$$

$$t^2 + t - 6 = 0$$

$$(t-2)(t+3) = 0$$

The normal meets the curve when t = 2 (point A) and when t = -3 (point B)

When t = -3, $x = 2(-3)^2 = 18$

$$y = 4 \times -3 = -12$$

The coordinates of B are (18, -12).

[5]

2. (i) $x = 3t \Rightarrow \dfrac{dx}{dt} = 3$

$$y = 12t - t^3 \Rightarrow \frac{dy}{dt} = 12 - 3t^2$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{12-3t^2}{3} = 4 - t^2$$

At turning points: $\dfrac{dy}{dx} = 0 \ \Rightarrow 4 - t^2 = 0$

$$\Rightarrow t = \pm 2$$

When t = 2: x = 6 and y = 16 $\Rightarrow$ there is a turning point at (6, 16).

Consider the sign of $\dfrac{dy}{dx}$ just before and after x = 6:

t = 1.9 $\Rightarrow x$ = 5.7 (just before) and $\dfrac{dy}{dx} = 0.39 > 0$

t = 2.1 $\Rightarrow x$ = 6.3 (just after) and $\dfrac{dy}{dx} = -0.41 < 0$

So (6, 16) is a maximum.

When t = -2: x = -6 and y = -16 $\Rightarrow$ there is a turning point at (-6, -16).

Consider the sign of $\dfrac{dy}{dx}$ just before and after x = -6:

t = -2.1 $\Rightarrow x$ = -6.3 (just before) and $\dfrac{dy}{dx} = -0.41 < 0$

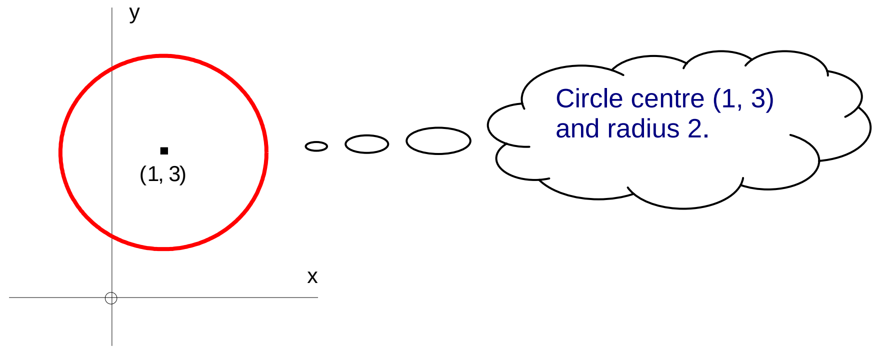
t = -1.9 $\Rightarrow x$ = -5.7 (just after) and $\dfrac{dy}{dx} = 0.39 > 0$

So (-6, -16) is a minimum.

[6]

Edexcel A level Maths Parametric Assessment solutions

3. (i) $x = 1 + 2\cos\theta,\ y = 3 + 2\sin\theta$



[2]

(ii) $x = 1 + 2\cos\theta \Rightarrow \dfrac{dx}{d\theta} = -2\sin\theta$

$y = 3 + 2\sin\theta \Rightarrow \dfrac{dy}{d\theta} = 2\cos\theta$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{dt/dt}{dx/dt} = \dfrac{\not{2}\cos\theta}{-\not{2}\sin\theta} = -\dfrac{\cos\theta}{\sin\theta} \quad (= -\cot\theta)$

[3]

(iii) Using $\quad y - y_1 = m(x - x_1)$ with $\quad x_1 = 1 + 2\cos\theta$

$y_1 = 3 + 2\sin\theta$

$m = -\dfrac{\cos\theta}{\sin\theta}$

Equation of tangent is $y - (3 + 2\sin\theta) = \dfrac{-\cos\theta}{\sin\theta}(x - (1 + 2\cos\theta))$

$\Rightarrow y\sin\theta - 3\sin\theta - 2\sin^2\theta = -\cos\theta(x - 1 - 2\cos\theta)$

$\Rightarrow (y - 3)\sin\theta - 2\sin^2\theta = (1 - x)\cos\theta + 2\cos^2\theta$

$\Rightarrow (y - 3)\sin\theta + (x - 1)\cos\theta = 2\sin^2\theta + 2\cos^2\theta$

$\Rightarrow (y - 3)\sin\theta + (x - 1)\cos\theta = 2$

[3]

(iv) When $\theta = \dfrac{\pi}{3}$: $x = 1 + 2\cos\left(\dfrac{\pi}{3}\right) = 1 + 2 \times \dfrac{1}{2} = 2$

$y = 3 + 2\sin\left(\dfrac{\pi}{3}\right) = 3 + 2 \times \dfrac{\sqrt{3}}{2} = 3 + \sqrt{3}$

So the coordinates are $(2, 3 + \sqrt{3})$.

[2]

(v) Gradient of tangent $= -\cot\theta$, so gradient of normal $= \tan\theta$ (using $m_1 m_2 = -1$).

Gradient of normal at $\theta = \dfrac{\pi}{3}$ is $\tan\dfrac{\pi}{3} = \sqrt{3}$.

Using $y - y_1 = m(x - x_1)$ with $x_1 = 2$

$y_1 = 3 + \sqrt{3}$

$m = \sqrt{3}$

Equation of normal is $y - (3 + \sqrt{3}) = \sqrt{3}(x - 2)$

$$y - 3 - \sqrt{3} - \sqrt{3}x + 2\sqrt{3} = 0$$

$$y - \sqrt{3}x - 3 + \sqrt{3} = 0$$

[4]

4. (i) $x = \cos 2t \Rightarrow \dfrac{dx}{dt} = -2\sin 2t$

$$y = \sin^2 t \Rightarrow \frac{dy}{dt} = 2\sin t \times \cos t = \sin 2t$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\sin 2t}{-2\sin 2t} = -\frac{1}{2}$$

[3]

(ii) $x = \cos 2t$

$y = \sin^2 t$

Use the trig identity: $\cos 2t \equiv 1 - 2\sin^2 t$

$$x = 1 - 2y$$

$$2y + x = 1$$

[3]

5. (i) When graph cuts y-axis, $x = 0 \Rightarrow 2\sqrt{t} = 0$

$$\Rightarrow t = 0$$

When $t = 0$, $y = 2$

So A is $(0, 2)$.

When graph cuts x-axis, $y = 0 \Rightarrow t^2 - 3t + 2 = 0$

$$\Rightarrow (t - 1)(t - 2) = 0$$

$$\Rightarrow t = 1 \text{ or } t = 2$$

When $t = 1$, $x = 2\sqrt{1} = 2$

When $t = 2$, $x = 2\sqrt{2}$

So B is $(2, 0)$ and C is $(2\sqrt{2}, 0)$.

[3]

(ii) $x = 2t^{\frac{1}{2}} \quad \Rightarrow \frac{dx}{dt} = 2 \times \frac{1}{2} t^{-\frac{1}{2}} = t^{-\frac{1}{2}}$

$$\begin{aligned}
\text{Area} &= \int_{t=0}^{t=1} y \frac{dx}{dt} dt \\
&= \int_0^1 (t^2 - 3t + 2) t^{-\frac{1}{2}} dt \\
&= \int_0^1 (t^{\frac{3}{2}} - 3t^{\frac{1}{2}} + 2t^{-\frac{1}{2}}) dt \\
&= \left[\tfrac{2}{5} t^{\frac{5}{2}} - 3 \times \tfrac{2}{3} t^{\frac{3}{2}} + 2 \times 2t^{\frac{1}{2}} \right]_0^1 \\
&= \tfrac{2}{5} - 2 + 4 - 0 \\
&= 2.4
\end{aligned}$$

[6]