

Topic assessment

1. (i) By considering turning points, show that  $x^3 - 3x^2 + 5 = 0$  has only one real root and that this root lies between  $-2$  and  $-1$ . [4]
 

(ii) Show that this root is  $-1.104$ , correct to 3 d.p. [2]
  
2. (i) By sketching the line  $y = x + 7$  and the curve  $y = \frac{1}{8}x^4$ , show that the equation  $x^4 - 8x - 56 = 0$  has two real roots. [3]
 

(ii) Show that the positive root lies between  $x = 2$  and  $x = 3$ . [2]

(iii) Use the iterative formula  $x_{n+1} = \sqrt[4]{8x_n + 56}$ , starting from  $x = 3$ , to find the value of the positive root correct to 2 decimal places. [4]
  
3. (i) Show that the equation  $e^x = x^3 - 1$  has a real root between  $x = 2$  and  $x = 3$ . [2]
 

(ii) Use the iterative formula  $x_{n+1} = \frac{e^{x_n} + 1}{x_n^2}$ , starting from  $x_0 = 2$ , to find two further approximations to the root. [4]

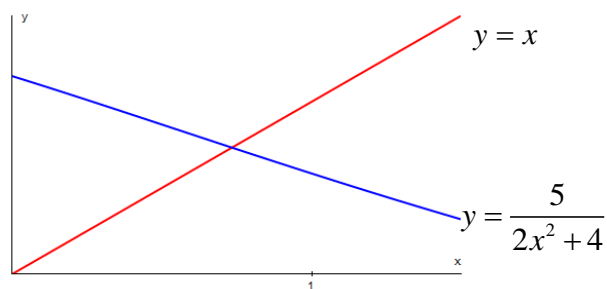
(iii) Show that the root is  $2.081$  correct to 3 decimal places. [2]
  
4. (i) Show that the gradient of  $y = 2x^3 + 4x - 5$  is always positive and deduce that the equation  $2x^3 + 4x - 5 = 0$  has one real root only. [2]
 

(ii) Show that this root lies between  $x = 0$  and  $x = 1$ . [2]

(iii) Show that the equation can be rearranged into the form  $x = \frac{5}{2x^2 + 4}$ . [2]

(iv) Using the iterative formula  $x_{n+1} = \frac{5}{2x_n^2 + 4}$  and starting from  $x_0 = 1$ , find the next two approximations  $x_1$  and  $x_2$  to the root. [4]

(v) The diagram below shows part of the graphs of  $y = x$  and  $y = \frac{5}{2x^2 + 4}$ , and the position of  $x_0$ .



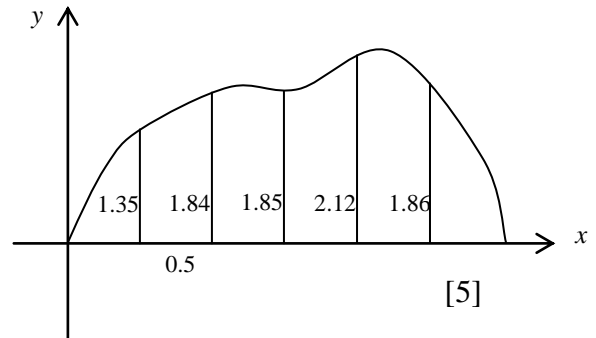
# Edexcel A level Maths Num methods Assessment

Copy the diagram and draw on it a staircase or cobweb diagram to illustrate how the iterations converge to the root. Indicate the positions of  $x_1$  and  $x_2$  on the  $x$ -axis. [2]

(vi) Show that the root is 0.893 correct to 3 decimal places. [2]

5. The root of the equation  $x^3 - x + 5 = 0$  is denoted by  $\alpha$ .  
Taking a first approximation  $x_1 = -2$ , use the Newton-Raphson method to find the value of  $\alpha$  correct to 4 decimal places. [6]

6. The diagram shows a cross-section of a tunnel. The height is measured in metres every 0.5 metres along the cross section. Use the trapezium rule to estimate the area of the cross-section.



Is it an under-estimate or over-estimate? [5]

7. An estimate is required for the integral  $\int_0^1 x\sqrt{x^3+1} dx$ .
- (i) Using 5 rectangles, find overestimates and underestimates for the value of the integral. [6]
- (ii) If 20 rectangles were used, find the difference between the overestimate and underestimate for the value of this integral. [3]
- (iii) The difference between the overestimate and the underestimate is required to be less than 0.001. Find the minimum number of rectangles required. [3]

**Total 60 marks**