Edexcel A level Mathematics Differentiation

Topic assessment

1. Using the chain rule, differentiate $(x^2 - 1)^6$. [3]

2. Show that the gradient of $y = (x^2 - 1)(x - 2)^3$ is given by
$$\frac{dy}{dx} = (x-2)^2(5x^2 - 4x - 3).$$ [4]

3. Find the gradient of the curve $y = \dfrac{x-1}{x^2 - 3}$ at the point where $x = 2$. [5]

4. A curve has equation $y = x^3 - 6x^2 + 1$.
Find the coordinates of the point of inflection. [4]

5. A potter is making an open topped vessel shaped as a right circular cylinder of radius r and height $2r$.
   (i) Find the rate at which the volume is increasing when the radius is 2 cm and increasing at a rate of 0.25 cm/s. [5]
   (ii) Given that the volume is increasing at a rate of 5π cm^3/s when the radius is 5 cm, find the rate at which the surface area is increasing at this point. [6]

6. A curve has equation $y = 3x^4 - 8x^3 + 6x^2 + 1$.
   (i) Find the coordinates of the stationary points and determine their nature. [6]
   (ii) Sketch the curve. [2]
   (iii) Find the values for x for which the curve is convex. [3]

7. Three pieces of wire are cut and used to make two equal circles and a square. The total length of wire used is 100 cm. If the radius of each circle is x cm and the side of the square y cm:
   (i) Write down an equation that connects x and y and simplify as far as possible. [3]
   (ii) Write down an expression for the total area enclosed (A) in terms of x and y. [2]
   (iii) Eliminate y from your expression in (ii) using a substitution from your equation in (i) and hence express A in terms of x only. [2]
   (iv) Find a value for x that will make A a minimum. [5]

Total 50 marks

Edexcel A level Maths Differentiation Assessment solns

Solutions to topic assessment

1. $y = (x^2 - 1)^6$

   Let $u = x^2 - 1 \quad \Rightarrow \dfrac{du}{dx} = 2x$

   $y = u^6 \quad \Rightarrow \dfrac{dy}{du} = 6u^5$

   Using the chain rule: $\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx} = 6u^5 \times 2x$

   $= 12x(x^2 - 1)^5$

   [3]

2. $y = (x^2 - 1)(x - 2)^3$

   Let $u = x^2 - 1 \quad \Rightarrow \dfrac{du}{dx} = 2x$

   Let $v = (x-2)^3 \quad \Rightarrow \dfrac{dv}{dx} = 3(x-2)^2$

   Using the product rule: $\dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$

   $= (x^2 - 1) \times 3(x-2)^2 + (x-2)^3 \times 2x$

   $= (x-2)^2\left[3(x^2 - 1) + 2x(x-2)\right]$

   $= (x-2)^2(3x^2 - 3 + 2x^2 - 4x)$

   $= (x-2)^2(5x^2 - 4x - 3)$

   [4]

3. $y = \dfrac{x-1}{x^2 - 3}$

   Let $u = x - 1 \quad \Rightarrow \dfrac{du}{dx} = 1$

   Let $v = x^2 - 3 \quad \Rightarrow \dfrac{dv}{dx} = 2x$

   Using the quotient rule: $\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

   $= \dfrac{(x^2 - 3) \times 1 - (x-1) \times 2x}{(x^2 - 3)^2}$

   $= \dfrac{x^2 - 3 - 2x^2 + 2x}{(x^2 - 3)^2}$

   $= \dfrac{-x^2 - 3 + 2x}{(x^2 - 3)^2}$

   When $x = 2$, gradient $= \dfrac{-2^2 - 3 + 2 \times 2}{(2^2 - 3)^2} = \dfrac{-4 - 3 + 4}{1^2} = -3$. [5]

4. $y = x^3 - 6x^2 + 1$

$\frac{dy}{dx} = 3x^2 - 12x$

$\frac{d^2y}{dx^2} = 6x - 12$

At point of inflection, $\frac{d^2y}{dx^2} = 0 \quad \Rightarrow x = 2$

When $x = 2, y = 8 - 24 + 1 = -15$

The point of inflection is (2, -15)

[4]

5. (i) $V = \pi r^2 h = \pi r^2 \times 2r = 2\pi r^3$

$\frac{dV}{dr} = 6\pi r^2$

Using the chain rule: $\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt} = 6\pi r^2 \frac{dr}{dt}$

When $r = 2$ and $\frac{dr}{dt} = 0.25$: $\frac{dV}{dt} = 6\pi \times 2^2 \times 0.25$

$= 6\pi$

$= 18.8 \text{ cm}^3/\text{s}$ (3 s.f.)

[5]

(ii) Surface area $A = 2\pi rh + \pi r^2$

$= 2\pi r \times 2r + \pi r^2$

$= 4\pi r^2 + \pi r^2$

$= 5\pi r^2$

$\frac{dA}{dr} = 10\pi r$

Using the chain rule: $\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dV} \times \frac{dV}{dt}$

$= 10\pi r \times \frac{1}{6\pi r^2} \frac{dV}{dt} = \frac{5}{3r} \frac{dV}{dt}$

When $r = 5$ and $\frac{dV}{dt} = 5\pi$, $\frac{dA}{dt} = \frac{5}{3 \times 5} \times 5\pi = \frac{5\pi}{3}$

$= 5.24 \text{ cm}^2/\text{s}$ (3 s.f.)

[6]

6. (i) $y = 3x^4 - 8x^3 + 6x^2 + 1$

$\frac{dy}{dx} = 12x^3 - 24x^2 + 12x$

At stationary points, $12x^3 - 24x^2 + 12x = 0$

$x(x^2 - 2x + 1) = 0$

$x(x-1)^2 = 0$

$x = 0$ or $x = 1$

When $x = 0$, $y = 1$

When $x = 1$, $y = 3 - 8 + 6 + 1 = 2$

$\frac{d^2y}{dx^2} = 36x^2 - 48x + 12$

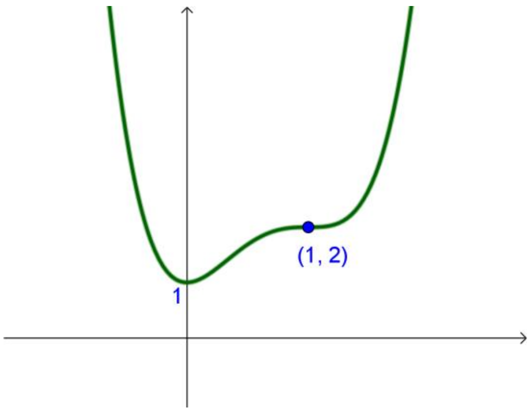
When $x = 0$, $\frac{d^2y}{dx^2} > 0$ so (0, 1) is a local minimum point.

When $x = 1$, $\frac{d^2y}{dx^2} = 0$

When $x = 0.5$, $\frac{dy}{dx} > 0$, and when $x = 2$, $\frac{dy}{dx} > 0$ so (1, 2) is a stationary point of inflection.

[6]

(ii)



[2]

(iii) The curve is convex where $\frac{d^2y}{dx^2} > 0$

$36x^2 - 48x + 12 > 0$

$3x^2 - 4x + 1 > 0$

$(x-1)(3x-1) > 0$

so it is convex for $x < \frac{1}{3}$ and $x > 1$.

[3]

7. (i) Wire used for square $= 4y$

Wire used for each circle $= 2\pi x$

Total length is 100 cm $\Rightarrow 4y + 4\pi x = 100$

$\Rightarrow y + \pi x = 25$

[3]

(ii) Area of square $= y^2$

Area of each circle $= \pi x^2$

Total area is given by $A = y^2 + 2\pi x^2$

[2]

(iii) From (i), $y = 25 - \pi x$

Substituting into expression in (ii): $A = (25 - \pi x)^2 + 2\pi x^2$

[2]

(iv) The expression for A is quadratic, with positive term in x^2, so the turning point is a minimum point.

$$\frac{dA}{dx} = 2(25 - \pi x) \times -\pi + 4\pi x$$

$$= -2\pi(25 - \pi x) + 4\pi x$$

At stationary point, $-2\cancel{\pi}(25 - \pi x) + 4\cancel{\pi}x = 0$

$$-25 + \pi x + 2x = 0$$

$$(2 + \pi)x = 25$$

$$x = \frac{25}{2 + \pi}$$

Therefore $x = \dfrac{25}{2 + \pi}$ minimises the value of A.

[5]