Summary sheet: Differential equations

G6 Construct simple differential equations in pure mathematics and in context,
(contexts may include kinematics, population growth and modelling the relationship between price and demand)

H7 Evaluate the analytical solution of simple first order differential equations with separable variables, including finding particular solutions
(Separation of variables may require factorisation involving a common factor)

H8 Interpret the solution of a differential equation in the context of solving a problem, including identifying limitations of the solution; includes links to kinematics

Differential equations

Differential equations include a derivative (e.g. $\frac{dy}{dx}$). If an equation includes a 1st derivative it is called a 1st order differential equation, if it includes a 2nd derivative it is called a 2nd order differential equation.

Constructing differential equations

Remember:

- Differential equations often involve rates of change. If 'something' is changing over time your derivative will be: $\frac{\mathrm{d}(something)}{\mathrm{d}t}$.
- If a is proportional to b you can write $a \propto b$ and therefore $a = kb$ (where k is the constant of proportionality)

e.g. construct a differential equation for the following scenario: the rate at which a plant grows is proportional to how much food it is given.

Let the amount of food = f
Rate of growth (g) = $\frac{\mathrm{d}g}{\mathrm{d}t}$

the rate at which a plant grows is proportional to how much food it is given $\therefore \frac{\mathrm{d}g}{\mathrm{d}t} = kf$

You have now constructed a 1st order differential equation.

Solving differential equations

To solve a differential equation, you need to integrate. If it is possible to use direct integration, then do. Also, see Integration summary sheet for the methods of substitution and integration by parts. Another method of integration is called **separating the variables**.

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Separating the variables

Think about direct integration: **e.g. find y when $\frac{dy}{dx} = x^2 + 3$**

You expect to see $x's$ on this side (the variable at the bottom of $\frac{dy}{dx}$) NOT $y's$.

If you see something like this: **find y when $\frac{dy}{dx} = y^2$** you will need to separate the variables.

To separate the variables

Check your notes to see why this works but an easy way to remember what to do is:

Collect any terms with a y on the left hand side
Collect any terms with an x on the right hand side

e.g. find y when $\frac{dy}{dx} = 2xy^2$

Collect $y's$ at the left and $x's$ at the right: $$\frac{dy}{y^2} = 2x\,dx$$

Rewrite as: $$y^{-2}dy = 2x\,dx$$

Include the integral sign on each side: $$\int y^{-2}dy = \int 2x\,dx$$

Integrate: $$-y^{-1} = \frac{2x^2}{2} + c$$

You only need the constant at one side.

Tidy up: $$-\frac{1}{y} = x^2 + c$$

$$y = -\frac{1}{x^2 + c}$$

To find the particular solution you need to be given more information.
e.g. for the above problem, you are told that when $x = 0$, $y = 2$.

Make c the subject: $$c = -x^2 - \frac{1}{y}$$

Insert given conditions: $$c = -0^2 - \frac{1}{2} \qquad \therefore c = -\frac{1}{2}$$

Substitute c in to find particular solution: $$y = -\frac{1}{x^2 - \frac{1}{2}}$$

You could tidy this up to: $$y = -\frac{2}{2x^2 - 1}$$