Summary sheet: Vectors

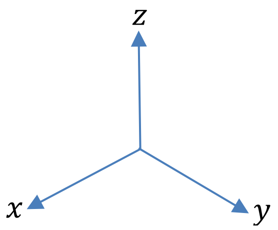
J1 Use vectors in three dimensions and three dimensions
J2 Calculate the magnitude and direction of a vector and convert between component form and magnitude/direction form
J3 Add vectors diagrammatically and perform the algebraic operations of vector addition and multiplication by scalars, and understand their geometrical interpretations
J4 Understand and use position vectors; calculate the distance between two points represented by position vectors
J5 Use vectors to solve problems in pure mathematics and in context, including forces

Vectors

A vector has both magnitude (size) and direction. To denote that something is a vector it can be written in bold (e.g. $\mathbf{u}$), underlined (e.g. $\underline{u}$) or with an arrow above (e.g. $\vec{u}$). Remember that $|\vec{a}|$ means the magnitude (size) of $\mathbf{a}$. See the AS summary sheet for more about vectors.

Vector forms

There are two different ways of writing a vector: **component form** and **magnitude/direction form**. Both forms tell you how to draw the vector but component form gives you the coordinates, and magnitude/direction tells you the angle to turn through and the length of the line. Now you are looking at 3D vectors you need to imagine the z-axis coming up out of the page.



This is the standard way to draw the axes. The unit vectors in the x, y and z directions are denoted by **i**, **j** and **k**.

Component form

Tells you how far to move along each axis. e.g. $\underline{u} = 3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$

Magnitude/direction form

Tells you the magnitude (length) of the line, and the angle turned through. This is more tricky with 3D vectors as it is not just the angle made with the x-axis that you are interested in. You need to know the angle made with the x-axis, the y-axis and the z-axis. You can find these angles by using the **direction cosines.**

For the position vector $\overrightarrow{OA}$:

The angle made with the x-axis (α): $\cos\alpha = \frac{x}{|\overrightarrow{OA}|}$

The angle made with the y-axis (β): $\cos\beta = \frac{y}{|\overrightarrow{OA}|}$

The angle made with the x-axis (τ): $\cos\tau = \frac{z}{|\overrightarrow{OA}|}$

These are called the **direction cosines**. Notice that the denominator is always the magnitude of the vector.

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e.g. Find the magnitude and the direction cosines for the vector $\overrightarrow{\mathbf{OA}} = \mathbf{2i - j + 4k}$:

Magnitude: $|\overrightarrow{\mathrm{OA}}| = \sqrt{2^2 + (-1)^2 + 4^2} = \sqrt{21}$

Direction cosines: $cos\ \alpha = \frac{2}{\sqrt{21}}$ $cos\ \beta = \frac{-1}{\sqrt{21}}$ $cos\ \tau = \frac{4}{\sqrt{21}}$

Addition and multiplication

To add vectors you add the $\mathbf{i}$, the $\mathbf{j}$ and $\mathbf{k}$ components separately:

e.g. add the vectors: $\underline{\mathbf{a}} = 3\mathbf{i} - 4\mathbf{j} + \mathbf{k}$ and $\underline{\mathbf{b}} = \mathbf{i} + \mathbf{j} - 5\mathbf{k}$

$$\underline{\mathbf{a}} + \underline{\mathbf{b}} = 4\mathbf{i} - 3\mathbf{j} - 4\mathbf{k}$$

To **multiply by a scalar**, multiply each component.

e.g. if $\underline{\mathbf{a}} = \mathbf{i} + 2\mathbf{j} - 4\mathbf{k}$ find $5\underline{\boldsymbol{a}}$

$$5\underline{\mathbf{a}} = 5(\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}) = 5\mathbf{i} + 10\mathbf{j} - 20\mathbf{k}$$

When multiplying by a scalar remember:

If you multiply by a +ve number (e.g. n): direction stays the same, size becomes n times bigger.

If you multiply by a -ve number (e.g. $-n$): direction reverses, size becomes n times bigger.

Position vectors

A position vector starts at the origin.

e.g. Point A $(3,\ 2, -5)$ has position vector $\overrightarrow{\mathrm{OA}} = 3\mathbf{i} + 2\mathbf{j} - 5\mathbf{k}$

For more about position vectors see AS summary sheet

Finding the distance between 2 points

This is the same as calculating the magnitude (size) of a vector (i.e. the distance between 2 points is the length of the line).

$$|\overrightarrow{\mathbf{AB}}| = \sqrt{(\boldsymbol{i_2 - i_1})^2 + (\boldsymbol{j_2 - j_1})^2 + (\boldsymbol{k_2 - k_1})^2}$$

e.g. Find the magnitude of $\overrightarrow{AB}$ if point A is at $(2, 5, -1)$ and point B is at $(3, -7, 2)$.

$$|\overrightarrow{\mathrm{AB}}| = \sqrt{(i_2 - i_1)^2 + (j_2 - j_1)^2 + (k_2 - k_1)^2}$$

$$|\overrightarrow{\mathrm{AB}}| = \sqrt{(3 - 2)^2 + (-7 - 5)^2 + (2 - -1)^2}$$

$$|\overrightarrow{\mathrm{AB}}| = \sqrt{(1)^2 + (-12)^2 + (3)^2}$$

$$|\overrightarrow{\mathrm{AB}}| = \sqrt{154}$$