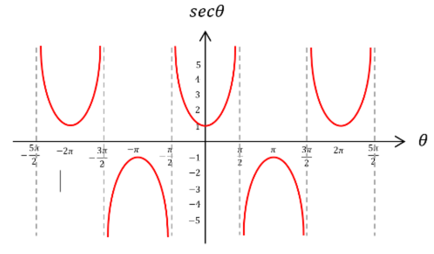
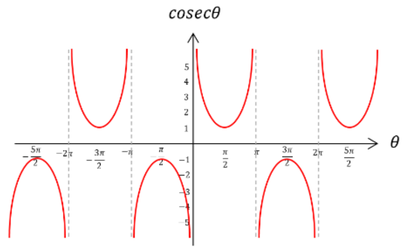
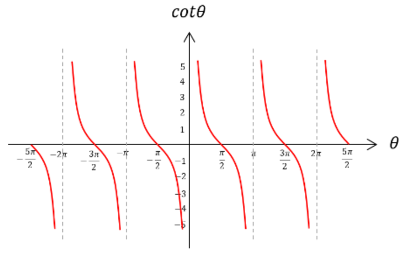
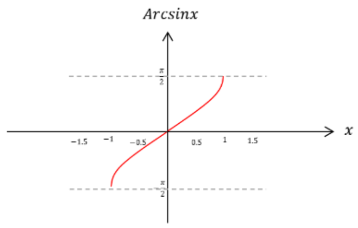
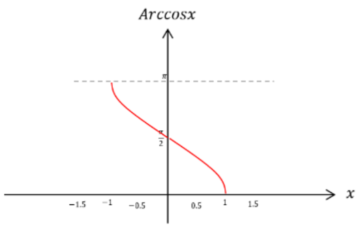
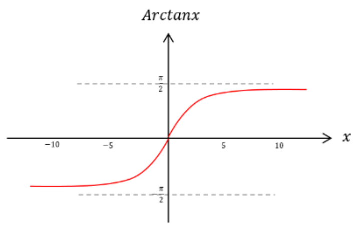
Summary sheet: Trigonometric functions

E4 Understand and use the definitions of secant, cosecant and cotangent and of arcsin, arccos and arctan; their relationships to sine, cosine and tangent; understanding of their graphs; their ranges and domains
E5 Understand and use; $\sec^2\theta = 1 + \tan^2\theta$ and $\text{cosec}^2\theta = 1 + \cot^2\theta$
E8 Construct proofs involving trigonometric functions and identities

Some definitions

Definition	Notes	Graph	Domain (D) & Range (R)
$\sec\theta = \dfrac{1}{\cos\theta}$	Remember that you cannot divide by 0, so whenever $\cos\theta, \sin\theta$ *or* $\tan\theta = 0$ the relevant function will be undefined. To sketch the graphs, sketch the original (e.g. $\cos\theta$) then invert it (like turning it inside out).		You can see from the graph that: D: $\theta \neq \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \ldots\ldots$ R: $\sec\theta \leq -1$ OR ≥ 1
$\text{cosec}\,\theta = \dfrac{1}{\sin\theta}$			You can see from the graph that: D: $\theta \neq 0, \pi, 2\pi, 3\pi \ldots\ldots$ R: $\text{cosec}\,\theta \leq -1$ OR ≥ 1
$\cot\theta = \dfrac{1}{\tan\theta}$			You can see from the graph that: D: $\theta \neq 0, \pi, 2\pi, 3\pi \ldots\ldots$ R: $-\infty < \text{cosec}\,\theta < \infty$

$\arcsin x = \sin^{-1}x$	Remember that these are the inverses and are used when you know the number and you want to find the angle. (e.g. $\sin\theta = 0.45$ so $\theta = \arcsin 0.45$)		You can see from the graph that: D: $-1 \leq x \leq 1$ R: $-\frac{\pi}{2} \leq \arcsin x \leq \frac{\pi}{2}$
$\arccos x = \cos^{-1}x$			You can see from the graph that: D: $-1 \leq x \leq 1$ R: $0 \leq \arccos x \leq \pi$
$\arctan x = \tan^{-1}x$			You can see from the graph that: D: $-\infty < x < \infty$ R: $-\frac{\pi}{2} < \arctan x < \frac{\pi}{2}$

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More identities

You should remember from previous work (see AS summary sheet) that:

$$\boxed{\sin^2\theta + \cos^2\theta = 1}$$

You can use this relationship to find 2 other ones that you need to know:

Start: $\sin^2\theta + \cos^2\theta = 1$

$\div$ *by* $\cos^2\theta$: $\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$

Tidy up: $\tan^2\theta + 1 = \sec^2\theta$ $\rightarrow$ $\boxed{\sec^2\theta = 1 + \tan^2\theta}$

Start: $\sin^2\theta + \cos^2\theta = 1$

$\div$ *by* $\sin^2\theta$: $\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$

Remember that: $\frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\tan^2\theta} = \cot^2\theta$

Tidy up: $1 + \cot^2\theta = \operatorname{cosec}^2\theta$ $\rightarrow$ $\boxed{\operatorname{cosec}^2\theta = 1 + \cot^2\theta}$