Summary sheet: Functions

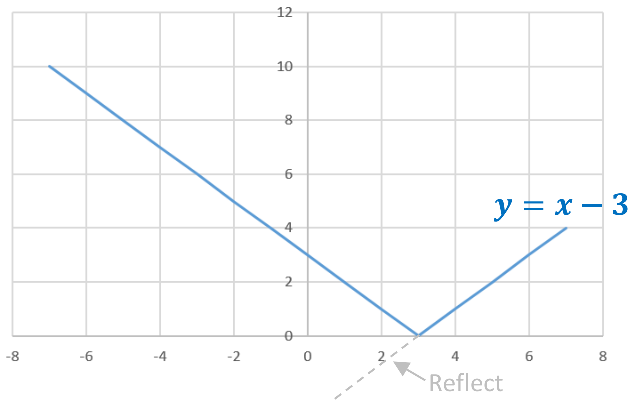
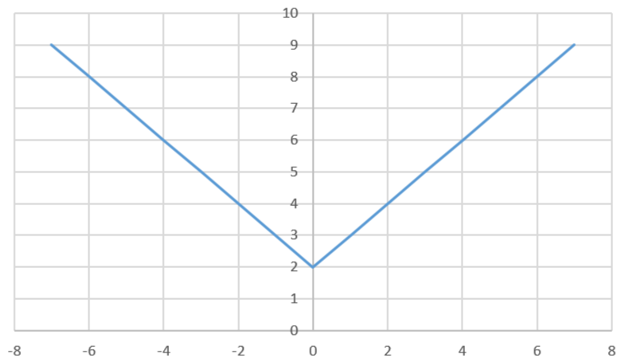
B7 Understand and use graphs of functions; sketch curves defined by simple equations including the modulus of a linear function
B8 Understand and use composite functions; inverse functions and their graphs
B9 Understand the effect of combinations of simple transformations on the graph of y = f (x).
B11 Use of functions in modelling, including consideration of limitations and refinements of the models

The modulus of a linear function

Remember that the modulus of a number means the absolute value (or "ignore the sign" because it is always positive). E.g. $|-5| = 5$

Sketching modulus graphs:

Sketch the graph <u>without</u> the modulus sign and then reflect the appropriate part.

Example:	What to do:	The graph:
$y = \|x - 3\|$	• Sketch $y = x - 3$ • The answer (y) cannot be negative so reflect the negative part in the x-axis.	
$y = \|x\| + 2$	• Sketch $y = x$ • Reflect the negative part in the x-axis • Translate the graph up 2.	

Functions

Remember that when dealing with function notation you can substitute the x for whatever you are trying to find, no matter how complicated.

e.g. if $\mathrm{f}(x) = 5x - 3$, find $\mathrm{f}(2)$ and $\mathrm{f}(x^2 + 1)$

$$\mathrm{f}(2) = 5(2) - 3 = 7$$

$$\mathrm{f}(x^2 + 1) = 5(x^2 + 1) - 3 = 5x^2 + 2$$

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Composite functions

Always apply the inside function first. e.g. to find $\mathrm{fg}(x)$, find $\mathrm{g}(x)$ first then substitute your answer into $\mathrm{f}(x)$ to find $\mathrm{f}(answer)$

e.g. If $\mathbf{f}(x) = \frac{3}{x-2}$ and $\mathbf{g}(x) = 2x - 4$ find $\mathbf{fg}(7)$ and $\mathbf{fg}(x)$

$$\mathrm{g}(7) = 2(7) - 4 = 10 \qquad \mathrm{f}(10) = \frac{3}{10-2} = \frac{3}{8}$$

$$\mathrm{g}(x) = 2x - 4 \qquad \mathrm{f}(2x-4) = \frac{3}{2x-4-2} = \frac{3}{2x-6}$$

Inverse functions

The inverse of $\mathrm{f}(x)$ is written $\mathrm{f}^{-1}(x)$. Remember that a function and its inverse 'undo' each other, so if you applied a function and then applied it's inverse you would be back where you started, i.e. $\mathrm{ff}^{-1}(x) = x$ and $\mathrm{f}^{-1}\mathrm{f}(x) = x$

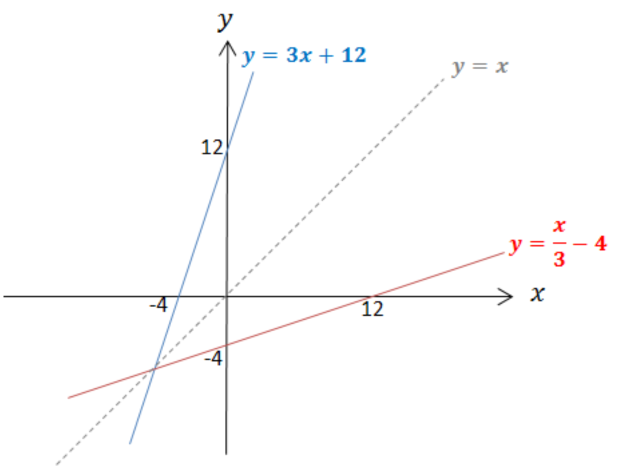
To find an inverse function:

Write as $y =$ Swap x and y Rearrange to make y the subject

e.g. if $\mathbf{f}(x) = 3x + 12$ find $\mathbf{f}^{-1}(x)$.

Write as $y =$	$y = 3x + 12$	
Swap x and y	$x = 3y + 12$	
Rearrange to make y the subject	$3y = x - 12$	
	$y = \frac{x}{3} - 4$	$\therefore \mathrm{f}^{-1}(x) = \frac{x}{3} - 4$

You can sketch the graphs of $\mathrm{f}(x)$ and $\mathrm{f}^{-1}(x)$ and you will see that they reflect in the line $y = x$.



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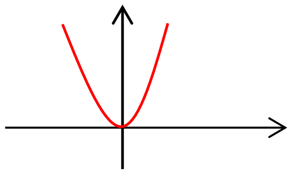
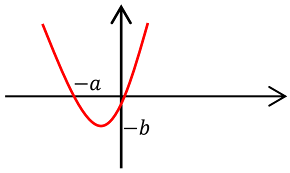
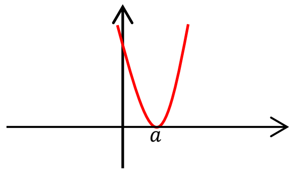
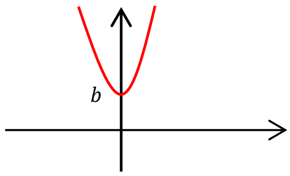
Transformations

See the AS summary sheet for single transformations. Transformations can also be combined, you need to think about what each part of the transformation would do then apply both (all) of them.

The following gives examples of some combination graph transformations:

e.g. graph: $\boldsymbol{y = x^2}$	Original	
$y = (x+a)^2 - b$	Translates left a and down b	
$y = b(x-a)^2$	Translates right a then stretches in the y-direction with factor b	
$y = ax^2 + b$	Stretches in the y-direction with factor a then translates up b	

Remember that these transformations work for all graphs, $y = x^2$ is just an example.
Try different graphs and combinations yourself to see what happens, and try replacing a and b with numbers to see how they affect your graphs.