Summary sheet: Sequences and series

D2 Work with sequences including those given by a formula for the nth term and those generated by a simple relation of the form $x_{n+1} = f(x_n)$; increasing sequences; decreasing sequences; periodic sequences D3 Understand and use sigma notation for sums of series D4 Understand and work with arithmetic sequences and series, including the formulae for nth term and the sum to n terms D5 Understand and work with geometric sequences and series including the formulae for the nth term and the sum of a finite geometric series; the sum to infinity of a convergent geometric series, including the use of $\|r\| < 1$; modulus notation D6 Use sequences and series in modelling

Sequences and series

A **sequence** is a set of numbers in a given order – they could form a pattern.
A **series** is the sum of the consecutive terms of a sequence.

Some types of sequences

Increasing:	Each number is bigger than the previous, i.e. $a_{n+1} > a_n$
Decreasing:	Each number is smaller than the previous, i.e. $a_{n+1} < a_n$
Periodic:	The sequence repeats (e.g. 1, 4, 7, 1, 4, 7, 1, 4, 7,…………)
Finite:	Has a first and a last term (it comes to an end).
Infinite:	No last term – continues to infinity.
Convergent:	Approaches a limit. You would use limit notation to denote the value it converges to. e.g. for the sequence 2.9, 2.99, 2.999, 2.9999, $\lim_{n\to\infty}(a_n) = 3$
Divergent:	Doesn't have a limit.

Some meanings

∑ (pronounced "sigma"): Means "the sum of"

e.g. $\sum_{k=1}^{k=4} 3^k$ means sum $3^1 + 3^2 + 3^3 + 3^4 = 120$

$\boldsymbol{x_{n+1} = \mathbf{f}(x_n)}$: Means "the next number in the sequence (x_{n+1}) is the function of the previous number $\left(f(x_n)\right)$"

e.g. Write down the first five terms of the sequence, $x_{n+1} = 3x_n + 4$ given that $x_0 = 1$
(i.e. to find the next term do 3 x previous term + 4)

$x_0 = 1$
$x_1 = 3(1) + 4 = 7$
$x_2 = 3(7) + 4 = 25$
$x_3 = 3(25) + 4 = 79$
$x_4 = 3(79) + 4 = 241$

The sequence is: 1, 7, 25, 79, 241

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Arithmetic sequences & series

An **arithmetic sequence** (sometimes called an arithmetic progression (AP)) has a **common difference** (i.e. the difference between consecutive terms is the same). E.g. 2, 5, 8, 11, 14, 17 is an arithmetic sequence with a common difference of 3. You need to be able to use the formulae to find any term in the sequence and also to sum together the terms of a sequence.

This can be used if you know the first and the last term.

Find the k_{th} term: $$\boldsymbol{a_k = a + (k-1)d}$$

Sum together the 1st n terms: $$\boldsymbol{S_n = \frac{1}{2}n[2a + (n-1)d]} \quad \text{OR} \quad \boldsymbol{S_n = \frac{1}{2}n(a + L)}$$

Where: a is 1st term, d is common difference and L is last term

e.g. for the sequence: 3, 7, 11, 15, 19……. Find the 12th term and sum of the first 8 terms.

You know that $a = 3$ and $d = 4$

$$a_{12} = 3 + (12-1)4 = 47$$
$$S_8 = \frac{1}{2} \times 8(2 \times 3 + (8-1)4) = 136$$

Geometric sequences & series

A **geometric sequence** (sometimes called a geometric progression (GP)) has a **common ratio** (i.e. you find the next term by multiplying the previous term by a fixed number) E.g. 2, 6, 18, 54, 162 is a geometric sequence with a common ratio of 3. You need to be able to use the formulae to find any term in the sequence and also to sum together the terms of the sequence.

Use whichever is easiest (e.g. the first one if $r < 1$ and the second one if $r > 1$)

Find the k_{th} term: $$\boldsymbol{a_k = ar^{k-1}}$$

Sum together the 1st n terms: $$\boldsymbol{S_n = \frac{a(1-r^n)}{1-r}} \quad \text{OR} \quad \boldsymbol{S_n = \frac{a(r^n-1)}{r-1}}$$

Sum to infinity: $$\boldsymbol{S_\infty = \frac{a}{1-r}}$$

For $-1 < r < 1$
i.e.: $|r| < 1$

N.B. You can only sum to infinity if $|r| < 1$ because if $|r| > 1$ the terms will get larger and larger and the series will not converge.

e.g. for the sequence: 1, 2, 4, 8, 16, 32 ……. Find the 15th term and sum of the first 10 terms.

You know that $a = 1$ and $r = 2$

$$a_{15} = 1 \times 2^{14} = 16384$$
$$S_{10} = \frac{1(2^{10}-1)}{2-1} = 1023$$