Edexcel A level Mathematics Further algebra

Section 1: The general binomial expansion

Section test

1. In the expansion of $(1+3x)^{-2}$, find
   - the coefficient of x^2
   - the coefficient of x^3
   - the values of x for which the expansion is valid.

2. In the expansion of $\dfrac{1}{3-2x}$, find
   - the coefficient of x
   - the coefficient of x^2
   - the values of x for which the expansion is valid.

3. In the expansion of $\dfrac{4-x}{\sqrt{1+2x}}$, find
   - the coefficient of x
   - the coefficient of x^2
   - the values of x for which the expansion is valid

4. An approximate value for $\sqrt{0.8}$ is found by substituting $x = 0.1$ into the first three terms of the binomial expansion for $\sqrt{1-2x}$.
   Find the percentage error in this approximate value.

Edexcel A level Maths Algebra 1 section test solutions

Section test solutions

1. $n = -2$

$$(1+3x)^{-2} = 1 + (-2)(3x) + \frac{-2 \times -3}{2!}(3x)^2 + \frac{-2 \times -3 \times -4}{3!}(3x)^3 + \ldots$$

$$= 1 - 6x + \left(3 \times 9x^2\right) - \left(6 \times 27x^3\right) + \ldots$$

$$= 1 - 6x + 27x^2 - 108x^3 + \ldots$$

The coefficient of x^2 is 27.

The coefficient of x^3 is -108.

The expansion is valid for: $-1 < 3x < 1$

$$-\tfrac{1}{3} < x < \tfrac{1}{3}$$

2. $(3-2x)^{-1} = \left(3^{-1}\right)\left(1 - \tfrac{2}{3}x\right)^{-1}$

$$= \tfrac{1}{3}\left(1 + (-1)\left(-\tfrac{2}{3}x\right) + \frac{-1 \times -2}{2!}\left(-\tfrac{2}{3}x\right)^2 + \ldots\right)$$

$$= \tfrac{1}{3}\left(1 + \tfrac{2}{3}x + \tfrac{4}{9}x^2 + \ldots\right)$$

$$= \tfrac{1}{3} + \tfrac{2}{9}x + \tfrac{4}{27}x^2 + \ldots$$

The coefficient of x is $\tfrac{2}{9}$.

The coefficient of x^2 is $\tfrac{4}{27}$.

The expansion is valid for: $-1 < -\tfrac{2}{3}x < 1$

$$-3 < -2x < 3$$

$$-\tfrac{3}{2} < x < \tfrac{3}{2}$$

3. $\dfrac{4 - x}{\sqrt{1 + 2x}} = (4 - x)(1 + 2x)^{-\frac{1}{2}}$

$$(1 + 2x)^{-\frac{1}{2}} = 1 + \left(-\tfrac{1}{2}\right)(2x) + \frac{-\tfrac{1}{2} \times -\tfrac{3}{2}}{2!}(2x)^2 + \ldots$$

$$= 1 - x + \tfrac{3}{8} \times 4x^2 + \ldots$$

$$= 1 - x + \tfrac{3}{2}x^2 + \ldots$$

$$(4 - x)(1 + 2x)^{-\frac{1}{2}} = (4 - x)\left(1 - x + \tfrac{3}{2}x^2 + \ldots\right)$$

$$= 4 - 4x + 6x^2 - x + x^2 - \tfrac{3}{2}x^3 + \ldots$$

$$= 4 - 5x + 7x^2 + \ldots$$

The coefficient of x is -5.

The coefficient of x^2 is 7.

This expansion is valid for: $-1 < 2x < 1$

$$-\tfrac{1}{2} < x < \tfrac{1}{2}$$

4. $(1-2x)^{\frac{1}{2}} = 1 + \left(\frac{1}{2}\right)(-2x) + \dfrac{\frac{1}{2} \times -\frac{1}{2}}{2!}(-2x)^2 + \ldots$

$\qquad = 1 - x - \frac{1}{8} \times 4x^2 + \ldots$

$\qquad = 1 - x - \frac{1}{2}x^2 + \ldots$

When $x = 0.1$: $(1 - 2 \times 0.1)^{\frac{1}{2}} = 1 - (0.1) - \frac{1}{2} \times (0.1)^2 + \ldots$

$\qquad\qquad\qquad \sqrt{0.8} \approx 0.895$

Percentage error $= \dfrac{\left|\text{Approximation } - \text{ exact value}\right|}{\text{exact value}} \times 100$

$\qquad\qquad = \dfrac{\left|0.895 - \sqrt{0.8}\right|}{\sqrt{0.8}} \times 100$

$\qquad\qquad = 0.064\% \ (3\text{d.p.})$