Section 3: Geometric sequences and series

Section test

1. For the geometric sequence 2, 6, 18, 54 …,
   What is the 8th term of the sequence?
   What is the sum of the first 10 terms of the sequence?
   What is the first term which is greater than 100000?

2. For the geometric sequence 24, -12, 6, -3 ….
   What is the 7th term of the sequence?
   What is the sum to infinity of the sequence?
   After how many terms is the sum of the sequence within 1% of the sum to infinity of the sequence?

3. A geometric sequence has 3rd term 36 and 5th term 81. Find the first term, *a*, and the common ratio, *r*.

4. A geometric sequence has first term 5 and sum to infinity 6.25. What is the common ratio of the sequence?

5. Anna's grandmother gives her £15 for her 10th birthday. She tells Anna that she will increase the amount she gives her for her birthday by £5 each year until she is 18.
   Amy's grandmother also gives her £15 for her 10th birthday. She tells Amy that she will increase the amount she gives her for her birthday by 25% each year until she is 18.

   On which birthday is Amy first given more than Anna?

   How much more money has Amy received in total than Anna by the time the girls are 18? (Give your answer to the nearest pound).

Edexcel A level Maths Series 3 section test solutions

Solutions to section test

1. First term $a = 2$
   Common ratio $r = 3$
   8^{th} term $= ar^7 = 2 \times 3^7 = 4374$

   $$S_{10} = \frac{a(r^{10} - 1)}{r - 1} = \frac{2(3^{10} - 1)}{3 - 1} = 59048$$

   n^{th} term $= ar^{n-1}$

   $$2 \times 3^{n-1} > 100000$$
   $$3^{n-1} > 50000$$
   $$(n-1)\log 3 > \log 50000$$
   $$n - 1 > 9.8$$
   $$n > 10.8$$

   The first term which is greater than 100000 is the 11^{th} term.

2. First term $a = 24$
   Common ratio $r = -0.5$
   7^{th} term $= ar^6 = 24 \times 0.5^6 = 0.375$

   $$S_\infty = \frac{a}{1 - r} = \frac{24}{1 - (-0.5)} = \frac{24}{1.5} = 16$$

   $$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{24(1 - (0.5)^n)}{1 - (-0.5)} = 16(1 - (0.5)^n)$$
   $$\left| \frac{16(1 - (-0.5)^n) - 16}{16} \right| < 0.01$$
   $$\left| -(-0.5)^n \right| < 0.01$$
   $$0.5^n < 0.01$$
   $$n \log 0.5 < \log 0.01$$
   $$n > \frac{\log 0.01}{\log 0.5}$$
   $$n > 6.6$$

   so the sum is first within 1% of the sum to infinity after 7 terms.

3. 3^{rd} term $= 36$ $\quad \Rightarrow ar^2 = 36 \quad (1)$
   5^{th} term $= 81$ $\quad \Rightarrow ar^4 = 81 \quad (2)$
   Dividing (2) by (1): $r^2 = \frac{81}{36} = \frac{9}{4} \quad \Rightarrow r = \frac{3}{2}$

Substituting into (1): $a\left(\frac{3}{2}\right)^2 = 36$

$$a = 36 \times \tfrac{4}{9} = 16$$

The first term is 16 and the common ratio is $\frac{3}{2}$.

4. $S_\infty = \dfrac{a}{1-r}$

$6.25 = \dfrac{5}{1-r}$

$1 - r = 0.8$

$r = 0.2$

The common ratio is 0.2.

5. Anna's presents form an arithmetic series with $a = 15$ and $d = 5$.
Amy's presents form a geometric series with $a = 15$ and $r = 1.25$.

10th birthday	Anna £15	Amy £15
11th birthday	Anna £20	Amy £18.75
12th birthday	Anna £25	Amy £23.48
13th birthday	Anna £30	Amy £29.30
14th birthday	Anna £35	Amy £36.62

Amy is first given more than Anna on the 14th birthday.

There are 9 birthdays from age 10 to 18.

Total for Anna $= \frac{1}{2}n[2a + (n-1)d] = \frac{1}{2} \times 9[30 + 8 \times 5] = £315$

Total for Amy $= \dfrac{a(r^n - 1)}{r - 1} = \dfrac{15(1.25^9 - 1)}{1.25 - 1} = £387$ to nearest pound

Amy has received £72 more than Anna.