Section 2: Arithmetic sequences and series

Section test

1. For the arithmetic sequence 2, 6, 10, …, 90
   What is the 12^{th} term of the sequence?
   What is the number of terms in the sequence?
   What is the sum of the terms of the sequence?

2. An arithmetic sequence has 25 terms. The first term is 60 and the last term is –12.
   Find the common difference.
   Find the sum of the terms of the sequence.

3. An arithmetic sequence has 4^{th} term 14 and 11^{th} term 70.
   Find the first term and the common difference.
   Find the sum of the first 12 terms of the sequence.

4. Sue goes swimming regularly. She wants to improve her fitness, so she decides to swim 10 lengths in the first session and increase the number of lengths she swims by 2 every session. When she reaches 50 lengths in a session she will not increase the number any further.
   After how many sessions does Sue swim 50 lengths for the first time?

   Sue decides she will give herself a reward when she has swum a total of 400 lengths. After how many sessions does she get her reward?

   Sue asks her friend Ming to come swimming with her. Ming starts coming at Sue's 8^{th} session. Ming starts by swimming 15 lengths and increases the number of lengths by 5 each time.
   After how many of Ming's sessions does she swim the same number of lengths as Sue?

Edexcel A level Maths Sequences 2 section test solns

Solutions to section test

1. The first term is 2 and the common difference is 4.

12^{th} term $= 2 + 11 \times 4 = 2 + 44 = 46$

n^{th} term $= 2 + 4(n-1)$

For the last term, $2 + 4(n-1) = 90$

$4(n-1) = 88$

$n - 1 = 22$

$n = 23$

$S_n = \frac{1}{2}n[a + l]$

$= \frac{1}{2} \times 23[2 + 90]$

$= \frac{1}{2} \times 23 \times 92$

$= 1058$

2. $a = 60, n = 25$

25^{th} term $= a + (n-1)d$

$= 60 + 24d$

$60 + 24d = -12$

$24d = -72$

$d = -3$

$a = 60, d = -3, n = 25$

$S_n = \frac{1}{2}n[2a + (n-1)d]$

$= \frac{1}{2} \times 25[2 \times 60 + 24 \times -3]$

$= \frac{1}{2} \times 25[120 - 72]$

$= \frac{1}{2} \times 25 \times 48$

$= 25 \times 24$

$= 600$

3. 4^{th} term $= a + 3d$, so $a + 3d = 14 \quad \Rightarrow a = 14 - 3d$

11^{th} term $= a + 10d$, so $a + 10d = 70$

Substituting first equation into second: $(14 - 3d) + 10d = 70$

$14 + 7d = 70$

$7d = 56$

$d = 8$

$a = 14 - 3d = 14 - 3 \times 8 = -10$

$S_n = \frac{1}{2}n[2a + (n-1)d]$

$S_{12} = \frac{1}{2} \times 12[(2 \times -10) + (11 \times 8)]$

$= 6[-20 + 88]$

$= 6 \times 68$

$= 408$

4. $a = 10, d = 2$

nth term $= 10 + 2(n-1)$

She swims 50 lengths after n sessions: $10 + 2(n-1) = 50$

$2(n-1) = 40$

$n - 1 = 20$

$n = 21$

She has swum 50 lengths for the first time after 21 sessions.

$a = 10, d = 2, S_n = 400$

$S_n = \frac{1}{2}n[2a + (n-1)d]$

$400 = \frac{1}{2}n[2 \times 10 + 2(n-1)]$

$400 = n[20 + 2n - 2]$

$400 = \frac{1}{2}n(2n + 18)$

$400 = n^2 + 9n$

$n^2 + 9n - 400 = 0$

$(n - 16)(n + 25) = 0$

Since n must be positive, $n = 16$.

She gets her reward after 16 sessions.

At Sue's 8th session she swims 24 lengths.

At Ming's nth session, Sue's lengths $= 24 + 2(n-1)$

At Ming's nth session, Ming's lengths $= 15 + 5(n-1)$

When they swim the same number, $24 + 2(n-1) = 15 + 5(n-1)$

$24 + 2n - 2 = 15 + 5n - 5$

$22 + 2n = 10 + 5n$

$12 = 3n$

$n = 4$