Edexcel A level Maths Sequences and series

Section 1: Definitions and notation

Section test

Questions 1 to 3 are about the following sequences:

A	2, 5, 8, 11, 14, …
B	3, 6, 12, 24, 48, …
C	1, 1, 2, 3, 5, 8, …
D	1, 3, 5, 3, 1, 3, …

1) Which of the above sequences is an arithmetic sequence?
Which of the above sequences is a geometric sequence?
Which of the above sequences could be a periodic sequence with period less than 5?

2) A sequence is defined by $a_{k+1} = 2a_k - 1,\ a_1 = 2$
Find the 6th term of this sequence.
Find $\sum_1^5 a_k$.

3) A sequence is defined by $a_k = k(k+1)$
Find the 5th term of this sequence.
Find $\sum_1^4 a_k$.

Questions 4 to 6 are about the sequence
1, -2, 4, -8, 16, -32

4) The sequence is defined inductively by
(a) $a_{k+1} = a_k - 2,\ a_1 = 1$
(b) $a_{k+1} = 2a_k,\ a_1 = 1$
(c) $a_{k+1} = a_k - 2^k,\ a_1 = 1$
(d) $a_{k+1} = -2a_k,\ a_1 = 1$

5) The sequence is defined deductively by
(a) $a_k = -2^{k-1}$
(b) $a_k = (-2)^{k-1}$
(c) $a_k = (-2)^k$
(d) $a_k = -2^k$

6) Find $\sum_4^6 a_k$.

Edexcel A level Maths Sequences 1 section test solns

Solutions to section test

1) The terms in sequence A go up by 3 each time, so A is an arithmetic sequence.

The terms in B are obtained by multiplying the previous term by 2, so B is a geometric sequence.

The 5th and 6th terms in sequence D are the same as the 1st and 2nd terms, so sequence D could be periodic with period 4.

2) $a_1 = 2$

$a_2 = 2a_1 - 1 = 2\times 2 - 1 = 3$

$a_3 = 2a_2 - 1 = 2\times 3 - 1 = 5$

$a_4 = 2a_3 - 1 = 2\times 5 - 1 = 9$

$a_5 = 2a_4 - 1 = 2\times 9 - 1 = 17$

$a_6 = 2a_5 - 1 = 2\times 17 - 1 = 33$

Using the terms from above,

$\sum_1^5 a_k = 2 + 3 + 5 + 9 + 17 = 36$.

3) Putting $k = 5$: $a_5 = 5(5+1) = 5\times 6 = 30$

$\sum_1^4 a_k = (1\times 2) + (2\times 3) + (3\times 4) + (4\times 5)$.

$= 2 + 6 + 12 + 20$

$= 40$

4) Each term is obtained by multiplying the previous term by -2, so the inductive definition is $a_{k+1} = -2a_k$, $a_1 = 1$.

5) Since each term is obtained by multiplying the previous term by -2, the deductive formula involves a power of -2. Since the first term is 1, the terms of the sequence are given by $(-2)^0, (-2)^1, (-2)^2, \ldots$

The deductive definition is therefore $a_k = (-2)^{k-1}$.

Notice that brackets are essential, otherwise all terms would be negative.

6) $\sum_4^6 a_k = a_4 + a_5 + a_6 = -8 + 16 - 32 = -24$