Section 1: Definitions and notation

Notes and Examples

In this section you will learn definitions and notation involving sequences and series, and some different ways in which sequences and series can be generated.

These notes contain subsections on

Types of sequence

A sequence is a set of numbers in a given order. These numbers may form an algebraic pattern.

Sequences can often be described in many ways. Here are some useful ways to describe a sequence.

- In an **increasing sequence**, each term is greater than the one before.
- In a **decreasing sequence**, each term is less than the one before.
- In an **arithmetic sequence**, the difference between one term and the next is always the same.
  For example: 2, 5, 8, 11, 14, …
  or 3, 2, 1, 0, -1, …
- In a **geometric sequence**, the ratio of one term to the next is always the same.
  For example: 1, 3, 9, 27, 81, …
  or 4, -2, 1, $-\frac{1}{2}$ …
- A **periodic sequence** repeats itself at regular intervals. The number of terms before the sequence repeats is called the **period**.
  So the sequence 1, 3, -4, 1, 3, -4, 1, 3, -4, … is periodic with period 3.

A series is the sum of the terms of a sequence. You need to be familiar with the Σ notation for a series (Σ is pronounced 'sigma' and is the Greek capital S):

$\sum_{1}^{10} a_k$ means the series $a_1 + a_2 + a_3 + \ldots + a_{10}$.

Sequences which follow a pattern can be defined algebraically in one of two ways: deductively or inductively.

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Sequences defined deductively

A deductive definition gives a direct formula for the kth term of the sequence in terms of k. The terms of the sequence can be found by substituting the numbers 1, 2, 3… for k.

Example 1

A sequence is defined deductively by

$$a_k = k^2 - 3$$

(i) Write down the first five terms of the sequence.

(ii) Find the 20th term of the sequence.

(iii) Find $\sum_{1}^{5} a_k$

Solution

(i) Substituting $k = 1$, $k = 2$, … $k = 5$ into the expression $a_k = k^2 - 3$ gives the sequence

–2, 1, 6, 13, 22

(ii) Substituting $k = 20$

$$\begin{aligned} a_{20} &= 20^2 - 3 \\ &= 400 - 3 \\ &= 397 \end{aligned}$$

(iii) $$\begin{aligned} \sum_{1}^{5} a_k &= -2 + 1 + 6 + 13 + 22 \\ &= 40 \end{aligned}$$

Sequences defined inductively

An inductive definition tells you how to find a term in a sequence from the previous term. The definition must also include the value of the first term of the sequence. You can then find the second term from the first term, the third term from the second term, and so on.

Example 2

A sequence is defined inductively as

$$a_{k+1} = 2a_k + 1, \ a_1 = 0$$

(i) Write down the first six terms of the sequence.

(ii) Find $\sum_{1}^{6} a_k$

Solution

(i) Each term is found by doubling the previous term and adding 1.
The first term is 0.

$a_1 = 0$
$a_2 = 2a_1 + 1 = 2 \times 0 + 1 = 1$
$a_3 = 2a_2 + 1 = 2 \times 1 + 1 = 3$
$a_4 = 2a_3 + 1 = 2 \times 3 + 1 = 7$
$a_5 = 2a_4 + 1 = 2 \times 7 + 1 = 15$
$a_6 = 2a_5 + 1 = 2 \times 15 + 1 = 31$
The first six terms are 0, 1, 3, 7, 15, 31

(ii) $\sum_{1}^{6} a_k = 0 + 1 + 3 + 7 + 15 + 31$
$= 57$