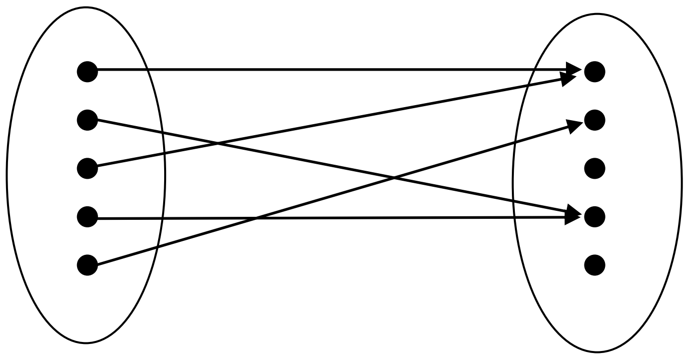
Section 1: Functions, graphs and transformations

Section test

1. The diagram below represents a mapping which is:



(a) one-to-one
(b) one-to-many
(c) many-to-one
(d) many-to-many

2. The function f is defined by:

   $f: x \rightarrow 1 - x^2$, where $-1 \leq x \leq 1$.

   What is the range of the function?

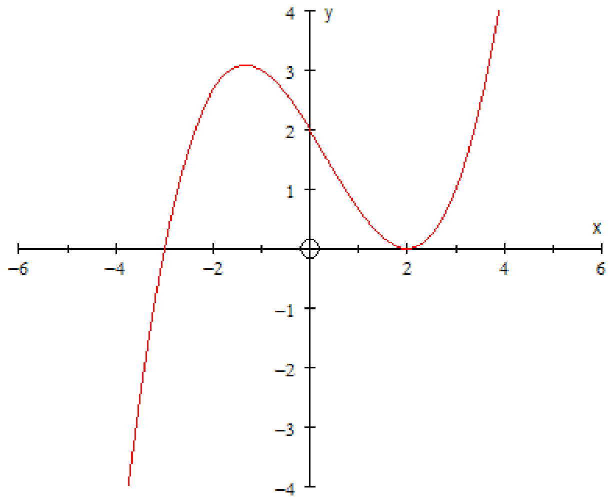
3. The function g is defined by:

   $g: x \rightarrow x^2 - x - 6, \; x \in \mathbb{R}$

   What is the value of g(–4)?

   When g(x) = 6, what are the possible values of x?

4. The graph below represents the function f(x).



   The graphs P, Q, R and S below represent various transformations of the function f(x).

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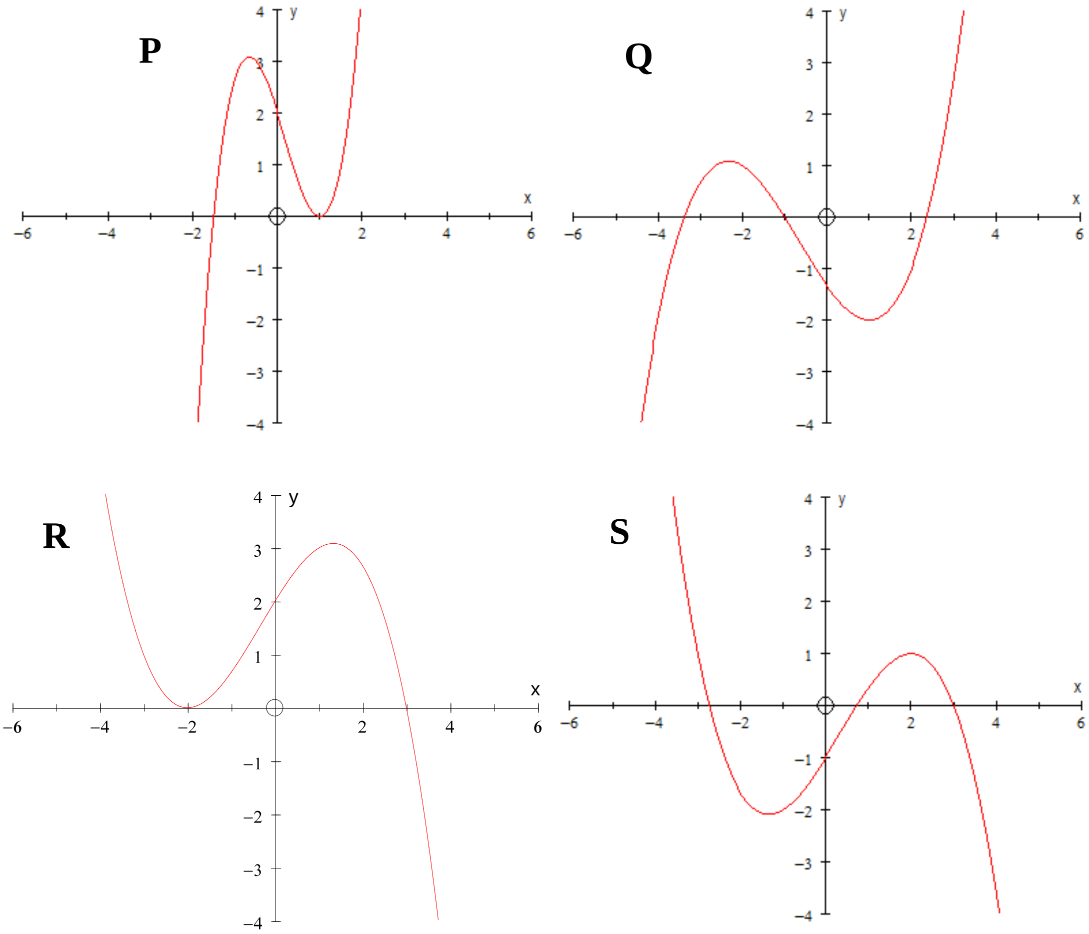

Which of the graphs represents the function $f(-x)$?
Which of the graphs represents the function $f(2x)$?
Which of the graphs represents the function $f(x+1)-2$?
Which of the graphs represents the function $1-f(x)$?

5. The graph of $y=\sin x$ is first translated 1 unit to the left, then stretched parallel to the x-axis with scale factor 2. The equation of the new graph is

(a) $y=\sin(2x+1)$ (b) $y=\sin(\frac{1}{2}x+1)$

(c) $y=\sin\frac{1}{2}(x+1)$ (d) $y=\sin 2(x+1)$

6. The graph of $y=\dfrac{1}{x}$ is first translated 2 units to the right, then reflected in the x-axis, then translated 1 unit vertically upwards. The equation of the new graph is

(a) $y=\dfrac{1}{x+2}+1$ (b) $y=1-\dfrac{1}{x+2}$

(c) $y=\dfrac{1}{x-2}+1$ (d) $y=1-\dfrac{1}{x-2}$

7. The graph of $y=x^2-2x+3$ is first reflected in the y axis, then translated 2 units vertically downwards, then stretched parallel to the y-axis with scale factor 3. What is the equation

of the new graph?

8. Describe the transformations required to obtain the graph of $y = (2x - 1)^2$ from the graph of $y = x^2$.

9. Describe the transformations required to obtain the graph of $y = 2\cos(x + 1) - 3$ from the graph of $y = \cos x$.

10. Describe the transformations required to obtain the graph of $y = 1 - (x + 2)^3$ from the graph of $y = x^3$.

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Solutions to section test

1. Each element in the domain is mapped to just one point in the co-domain, but some elements in the co-domain are images of more than one point in the domain. So this mapping is many-to-one.

2. The smallest possible value of f(x) where $-1 \le x \le 1$ is 0, when $x = \pm 1$.
   The greatest possible value of f(x) where $-1 \le x \le 1$ is 1, when $x = 0$.
   So the range of the function is given by y: $0 \le y \le 1$.

3. $g(x) = x^2 - x - 6$

$$g(-4) = (-4)^2 - (-4) - 6$$
$$= 16 + 4 - 6$$
$$= 14$$

$$g(x) = x^2 - x - 6$$
$$x^2 - x - 6 = 6$$
$$x^2 - x - 12 = 0$$
$$(x-4)(x+3) = 0$$
$$x = 4 \text{ or } x = -3$$

4. The graph of $y = f(-x)$ is obtained from the graph of $y = f(x)$ by a reflection in the y-axis. This is graph R.

   The graph of $y = f(2x)$ is obtained from the graph of $y = f(x)$ by a stretch of scale factor $\frac{1}{2}$ parallel to the x-axis. This is graph P.

   The graph of $y = f(x+1) - 2$ is obtained from the graph of $y = f(x)$ by a translation through $\begin{pmatrix} -1 \\ -2 \end{pmatrix}$. This is graph Q.

   The graph of $y = 1 - f(x)$ is obtained from the graph of $y = f(x)$ by a reflection in the x-axis and a translation of 1 unit vertically upwards. This is graph S.

5. $y = \sin x$
   Translating 1 unit to the left means f(x) becomes f(x + 1).
   This gives $y = \sin(x+1)$.
   A stretch parallel to the x-axis with scale factor 2 means f(x) becomes $f(\frac{1}{2}x)$.
   This gives $y = \sin(\frac{1}{2}x + 1)$

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6. $y = \frac{1}{x}$

   Translation of 2 units to the right means f(x) becomes f(x – 2).

   This gives $y = \frac{1}{x-2}$.

   Reflection in the x-axis means f(x) becomes –f(x)

   This gives $y = -\frac{1}{x-2}$

   Translation of 1 unit vertically upwards means f(x) becomes f(x) + 1.

   This gives $y = -\frac{1}{x-2} + 1$, or $y = 1 - \frac{1}{x-2}$.

7. $y = x^2 - 2x + 3$

   Reflection in the y-axis means f(x) becomes f(-x).

   This gives $y = (-x)^2 - 2(-x) + 3 = x^2 + 2x + 3$.

   Translation of 2 units vertically downwards means f(-x) becomes f(-x) – 2.

   This gives $y = x^2 + 2x + 3 - 2 = x^2 + 2x + 1$.

   Stretched parallel to the y-axis with scale factor 3 means f(-x) – 2 becomes 3(f(-x) – 2).

   This gives $y = 3\left(x^2 + 2x + 1\right) = 3x^2 + 6x + 3$.

8. Start with $y = x^2$

   Replace x with (x – 1) to give $y = (x-1)^2$. This is a horizontal translation of 1 unit to the right.

   Replace x with 2x to give $y = (2x-1)^2$. This is a stretch parallel to the x-axis, scale factor $\frac{1}{2}$.

9. Start with $y = \cos x$.

   Replace x with (x + 1) to give $y = \cos(x+1)$. This is a horizontal translation of 1 unit to the left .

   Replace f(x) with 2f(x) to give $y = 2\cos(x+1)$. This is a stretch of scale factor 2 parallel to the y-axis.

   Replace f(x) with f(x) – 3 to give $y = 2\cos(x+1) - 3$. This is a vertical translation of 3 units downwards.

10. Start with $y = x^3$.

    Replace x with (x + 2) to give $y = (x+2)^3$. This is a horizontal translation of 2 units to the left.

    Replace f(x) with –f(x) to give $y = -(x+2)^3$. This is a reflection in the x-axis.

    Replace f(x) with f(x) + 1 to give $y = 1 - (x+2)^3$. This is a vertical translation of 1 unit upwards.