# Section 2: Composite and inverse functions

## Section test

Questions 1 – 4 are about the functions f, g and h which are defined as follows:

$\mathrm{f}(x)=\sqrt{x-2} \quad x\geq 2$

$\mathrm{g}(x)=2x-3 \quad x\in\mathbb{R}$

$\mathrm{h}(x)=x^2 \quad x\geq 0$

1. The composite function fg is defined as:

(a) $\mathrm{fg}(x)=\sqrt{2x-3}$

(b) $\mathrm{fg}(x)=2\sqrt{x-2}-3$

(c) $\mathrm{fg}(x)=(2x-3)\sqrt{x-2}$

(d) $\mathrm{fg}(x)=\sqrt{2x-5}$

2. The composite function hg is defined as:

(a) $\mathrm{hg}(x)=2x^2-3$

(b) $\mathrm{hg}(x)=(2x-3)^2$

(c) $\mathrm{hg}(x)=x^2(2x-3)$

(d) $\mathrm{hg}(x)=(2x)^2-3$

3. The inverse of the function g is given by:

(a) $\mathrm{g}^{-1}(x)=\dfrac{x+3}{2},\quad x\in\mathbb{R}$

(b) $\mathrm{g}^{-1}(x)=\dfrac{x}{2}+3,\quad x\in\mathbb{R}$

(c) $\mathrm{g}^{-1}(x)=2x+3,\quad x\in\mathbb{R}$

(d) $\mathrm{g}^{-1}(x)=\dfrac{x}{2}-3,\quad x\in\mathbb{R}$

4. The inverse of the function f is given by:

(a) $\mathrm{f}^{-1}(x)=(x+2)^2,\quad x\geq 2$

(b) $\mathrm{f}^{-1}(x)=(x+2)^2,\quad x\geq 0$

(c) $\mathrm{f}^{-1}(x)=x^2+2,\quad x\geq 2$

(d) $\mathrm{f}^{-1}(x)=x^2+2,\quad x\geq 0$

 

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5. The functions p, q and r are given by:

$$\mathrm{p}(x) = ax$$
$$\mathrm{q}(x) = x + b$$
$$\mathrm{r}(x) = \frac{1}{x}$$

Write the function $\frac{1}{a(x+b)}$ as a composite function in terms of p, q and r.

Write the function $\frac{a}{x} + b$ as a composite function in terms of p, q and r.

Write the function $a^2x + 2b$ as a composite function in terms of p and q.

6. Which one of the functions below does **not** have an inverse function?

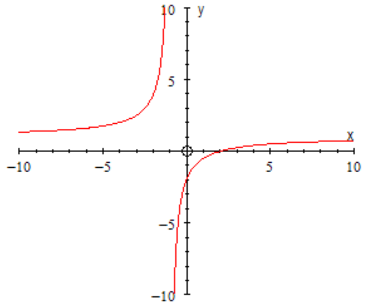
(a) $\mathrm{f}(x) = \frac{1}{x-1}$

(b) $\mathrm{f}(x) = x^3 + 1$

(c) $\mathrm{f}(x) = x^2 + 3, \quad x \geq 0$

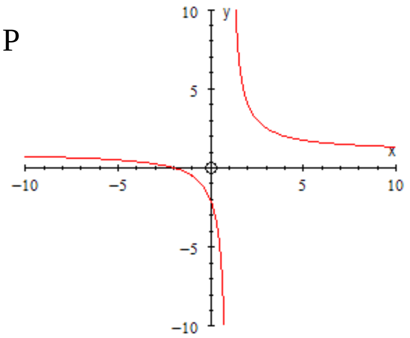
(d) $\mathrm{f}(x) = x^3 - 2x$

7. The graph of a function f($x$) is shown below.

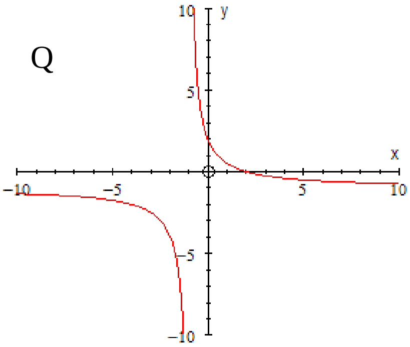



Which one of the four graphs P, Q, R and S shows the inverse function $\mathrm{f}^{-1}(x)$?

P




Q



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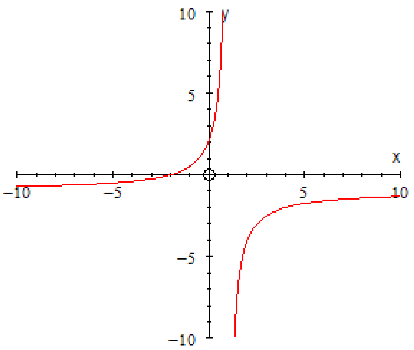
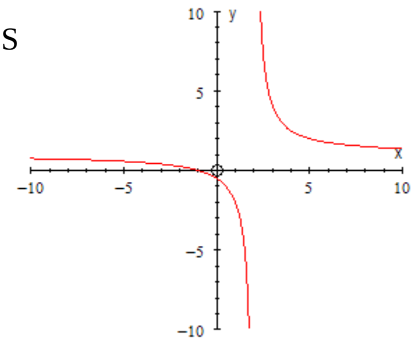
R




S




8. The inverse of the function $\dfrac{3x+1}{2-x}$ is:

(a) $\dfrac{2+x}{3x-1}$

(b) $\dfrac{2-x}{3x+1}$

(c) $\dfrac{2x+1}{x-3}$

(d) $\dfrac{2x-1}{x+3}$

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## Solutions to section test

1. $fg(x) = f(2x-3)$

$$= \sqrt{(2x-3)-2}$$
$$= \sqrt{2x-5}$$

2. $hg(x) = h(2x-3)$

$$= (2x-3)^2$$

3. $y = 2x - 3$

Interchanging x and y: $x = 2y - 3$

$$x + 3 = 2y$$
$$y = \frac{x+3}{2}$$

The range of g is $x \in \mathbb{R}$, so the domain of $g^{-1}$ is $x \in \mathbb{R}$.

The inverse of g is given by $g^{-1}(x) = \dfrac{x+3}{2}$.

4. $y = \sqrt{x-2}$

Interchanging x and y: $x = \sqrt{y-2}$

$$x^2 = y - 2$$
$$y = x^2 + 2$$

The range of f is $f(x) \geq 0$, so the domain of $f^{-1}$ is $x \geq 0$.

The inverse of f is given by $f^{-1}(x) = x^2 + 2, \quad x \geq 0$

5. The function $\dfrac{1}{a(x+b)}$ is obtained by first adding b, then multiplying by a, then taking the reciprocal. So the functions are applied in the order q, p, r, and therefore the function is rpq.

The function $\dfrac{a}{x} + b$ is obtained by first taking the reciprocal, then multiplying by a, then adding b. So the functions are applied in the order r, p, q, and therefore the function is qpr.

The function $a^2x + 2b$ is obtained by first multiplying by $a^2$, and then adding 2b. So the functions are applied in the order p, p, q, q, and therefore the function is $q^2p^2$.

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6. $f(x) = \dfrac{1}{x-1}$ and $f(x) = x^3 + 1$ are both one-to-one functions, so have inverses.

   The function $f(x) = x^2 + 3$ is not one-to-one over the real numbers, but over the given domain $x \geq 0$ it is one-to-one, so has an inverse.

   The function $f(x) = x^3 - 2x$ is not one-to-one over the real numbers, so it does not have an inverse.

7. The graph of the inverse function $y = f^{-1}(x)$ is the reflection of the graph of $y = f(x)$ in the line $y = x$. This is graph R.

8. $y = \dfrac{3x+1}{2-x}$

   Interchanging x and y: $x = \dfrac{3y+1}{2-y}$

   $$x(2-y) = 3y+1$$
   $$2x - xy = 3y+1$$
   $$2x - 1 = 3y + xy$$
   $$2x - 1 = y(3+x)$$
   $$y = \frac{2x-1}{x+3}$$

   The inverse of the function is $\dfrac{2x-1}{x+3}$.