Section 3: Partial fractions

Section test

1) For $\frac{13x-1}{(1-4x)(2+x)} \equiv \frac{A}{1-4x} + \frac{B}{2+x}$, find the values of the constants A and B.

2) Express $\frac{3x-28}{x^2-16}$ as the sum of partial fractions.

3) Express $\frac{7x+10}{(1-2x)(x+4)}$ as the sum of partial fractions.

4) Express $\frac{x^2+1}{(x-1)(x-2)(x+3)}$ as the sum of partial fractions.

5) Express $\frac{10x+1}{(2x+1)^2}$ as the sum of partial fractions.

6) For $\frac{2+5x-x^2}{x^2(x+1)} \equiv \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1}$, find the values of the constants A, B and C.

7) Express $\frac{3}{(2x-1)(x+1)^2}$ as the sum of partial fractions.

8) Find the first two terms of the expansion of $\frac{5}{(x-3)(x+2)}$.

9) Find the first three terms of the expansion of $\frac{3}{(1+x)(2x-1)}$.

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Section test solutions

1. $\dfrac{13x-1}{(1-4x)(2+x)} \equiv \dfrac{A}{1-4x} + \dfrac{B}{2+x}$

$13x-1 \equiv A(2+x)+B(1-4x)$

When $x=-2$: $-27 = 9B$

$B=-3$

When $x=\frac{1}{4}$: $\frac{9}{4} = \frac{9}{4}A$

$A=1$

$\dfrac{13x-1}{(1-4x)(2+x)} \equiv \dfrac{1}{1-4x} - \dfrac{3}{2+x}$

2. $\dfrac{3x-28}{x^2-16} \equiv \dfrac{3x-28}{(x+4)(x-4)} \equiv \dfrac{A}{(x+4)} + \dfrac{B}{(x-4)}$

$3x-28 \equiv A(x-4)+B(x+4)$

When $x=4$: $-16 = 8B$

$B=-2$

When $x=-4$: $-40 = -8A$

$A=5$

$\dfrac{3x-28}{x^2-16} \equiv \dfrac{5}{(x+4)} - \dfrac{2}{(x-4)}$

3. $\dfrac{7x+10}{(1-2x)(x+4)} \equiv \dfrac{A}{(1-2x)} + \dfrac{B}{(x+4)}$

$7x+10 \equiv A(x+4)+B(1-2x)$

When $x=-4$: $-18 = 9B$

$B=-2$

When $x=\frac{1}{2}$: $\frac{27}{2} = \frac{9}{2}A$

$A=3$

$\dfrac{7x+10}{(1-2x)(x+4)} \equiv \dfrac{3}{(1-2x)} - \dfrac{2}{(x+4)}$

4. $\dfrac{x^2+1}{(x-1)(x-2)(x+3)} \equiv \dfrac{A}{x-1} + \dfrac{B}{x-2} + \dfrac{C}{x+3}$

$x^2+1 \equiv A(x-2)(x+3)+B(x-1)(x+3)+C(x-1)(x-2)$

Putting $x=1 \quad \Rightarrow 2=-4A \quad \Rightarrow A=-\frac{1}{2}$

Putting $x=2 \quad \Rightarrow 5=5B \quad \Rightarrow B=1$

Equating coefficients of $x^2 \quad \Rightarrow 1=A+B+C \quad \Rightarrow C=\frac{1}{2}$

$\dfrac{x^2+1}{(x-1)(x-2)(x+3)} \equiv \dfrac{1}{x-2} + \dfrac{1}{2(x+3)} - \dfrac{1}{2(x-1)}$

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5. $\dfrac{10x+1}{(2x+1)^2} \equiv \dfrac{A}{(2x+1)} + \dfrac{B}{(2x+1)^2}$

$10x+1 \equiv A(2x+1)+B$

When $x=-\frac{1}{2}$: $\quad -4 = B$

By equating the constants: $1 = A+B$

$A = 5$

$\dfrac{10x+1}{(2x+1)^2} \equiv \dfrac{5}{(2x+1)} - \dfrac{4}{(2x+1)^2}$

6. $\dfrac{2+5x-x^2}{x^2(x+1)} \equiv \dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x+1}$

$2+5x-x^2 \equiv A(x)(x+1)+B(x+1)+C(x^2)$

When $x=0$: $\quad 2 = B$

When $x=-1$: $\quad -4 = C$

By equating the coefficients of x^2: $\quad -1 = A+C$

$A = 3$

$\dfrac{2+5x-x^2}{x^2(x+1)} \equiv \dfrac{3}{x} + \dfrac{2}{x^2} - \dfrac{4}{x+1}$

7. $\dfrac{3}{(2x-1)(x+1)^2} \equiv \dfrac{A}{(2x-1)} + \dfrac{B}{(x+1)} + \dfrac{C}{(x+1)^2}$

$3 \equiv A(x+1)^2 + B(x+1)(2x-1) + C(2x-1)$

When $x=-1$: $\quad 3 = -3C$

$C = -1$

When $x=\frac{1}{2}$: $\quad 3 = \frac{9}{4}A$

$A = \frac{4}{3}$

By equating the constants: $\quad 3 = A - B - C$

$B = -\frac{2}{3}$

$\dfrac{3}{(2x-1)(x+1)^2} \equiv \dfrac{4}{3(2x-1)} - \dfrac{2}{3(x+1)} - \dfrac{1}{(x+1)^2}$

8. $\dfrac{5}{(x-3)(x+2)} \equiv \dfrac{A}{(x-3)} + \dfrac{B}{(x+2)}$

$5 \equiv A(x+2)+B(x-3)$

When $x=3$: $\quad 5 \equiv 5A$

$A \equiv 1$

When $x=-2$: $\quad 5 \equiv -5B$

$B \equiv -1$

$\dfrac{5}{(x-3)(x+2)} \equiv \dfrac{1}{(x-3)} - \dfrac{1}{(x+2)} \equiv (x-3)^{-1} - (x+2)^{-1}$

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$$(x-3)^{-1} = \left(-3\left(1-\tfrac{1}{3}x\right)\right)^{-1} = \left(-3^{-1}\right)\left(1-\tfrac{1}{3}x\right)^{-1}$$
$$= -\tfrac{1}{3}\left(1+(-1)\left(-\tfrac{1}{3}x\right)+\ldots\right)$$
$$= -\tfrac{1}{3}\left(1+\tfrac{1}{3}x+\ldots\right)$$
$$= -\tfrac{1}{3}-\tfrac{1}{9}x+\ldots$$

$$(x+2)^{-1} = \left(2\left(1+\tfrac{1}{2}x\right)\right)^{-1} = \left(2^{-1}\right)\left(1+\tfrac{1}{2}x\right)^{-1}$$
$$= \tfrac{1}{2}\left(1+(-1)\left(\tfrac{1}{2}x\right)+\ldots\right)$$
$$= \tfrac{1}{2}\left(1-\tfrac{1}{2}x+\ldots\right)$$
$$= \tfrac{1}{2}-\tfrac{1}{4}x+\ldots$$
$$\frac{5}{(x-3)(x+2)} \equiv \left(-\tfrac{1}{3}-\tfrac{1}{9}x+\ldots\right)-\left(\tfrac{1}{2}-\tfrac{1}{4}x+\ldots\right)$$
$$\equiv -\tfrac{5}{6}+\tfrac{5}{36}x+\ldots$$

9. $$\frac{3}{(1+x)(2x-1)} \equiv \frac{A}{(1+x)}+\frac{B}{(2x-1)}$$
$$3 \equiv A(2x-1)+B(1+x)$$
When $x=-1$: $3=-3A$
$$A=-1$$
When $x=\tfrac{1}{2}$: $3=\tfrac{3}{2}B$
$$B=2$$
$$\frac{3}{(1+x)(2x-1)} \equiv \frac{2}{(2x-1)}-\frac{1}{(1+x)} \equiv 2(2x-1)^{-1}-(1+x)^{-1}$$
$$2(2x-1)^{-1} = -2(1-2x)^{-1}$$
$$= -2\left(1+(-1)(-2x)+\frac{-1\times-2}{2!}(-2x)^2+\ldots\right)$$
$$= -2\left(1+2x+4x^2+\ldots\right)$$
$$= -2-4x-8x^2+\ldots$$
$$(1+x)^{-1} = 1+(-1)(x)+\frac{-1\times-2}{2!}(x)^2+\ldots$$
$$= 1-x+x^2+\ldots$$
$$\frac{3}{(1+x)(2x-1)} \equiv \left(-2-4x-8x^2+\ldots\right)-\left(1-x+x^2+\ldots\right)$$
$$\equiv -3-3x-9x^2+\ldots$$