**2.5 Express solutions (to inequalities) through   
 set notation**

You have solved inequalities before and you will now learn to express the solution in set notation:

{*x* : … }

This may look complicated, but it simply means ‘all the values of *x* so that …’ or ‘the set of all *x* such that …’.

For example {*x* : *x* < 5} means all the values of *x* such that *x* is less than 5.

You may also need to use the signs 

* A  B means all the elements that are in set A **and** in set B.
* A  B means all the elements that are in set A **together with** all the elements that are in set B.

For example, if A = {*x* : *x* is odd}, B = {*x* : *x* < 4} and C = {*x* : *x* ≥ 9}   
then:

A  B is the values of *x* that are odd **and**less than 4

A  C is the values of *x* that are odd **and** greater than or equal to 9

B  C is the values of *x* that are less than 4 **together with** the values of *x* that are   
 greater or equal to 9

Remember that for the quadratic curve 

* if *a* is positive, the curve looks like this
* if *a* is negative the curve looks like this

A quadratic inequality is of the form   
  or .

* If  > 0 or  ≥ 0   
  you want the values of *x* where the curve is   
  **above** the *x*-axis.
* If < 0 or ≤ 0   
  you want the values of *x* where the curve is  
  **below** the *x*-axis.

The  signs are often used when solving quadratic inequalities.

Examples

**Example 1** Solve 3*x* + 2 > 5.   
Give your answer using set notation

|  |  |
| --- | --- |
| 3*x* + 2 > 5  3*x* > 3  *x* > 1  In set notation: {*x* : *x* > 1} | Solve the inequality in the usual way.  This is nearly the answer, but it does not use set notation.  This set notation means ‘all the values of *x* so that *x* > 1’ |

**Example 2** Solve *x*2 – 6*x* + 5 ≤ 0.  
 Give your answer using set notation.

|  |  |
| --- | --- |
| *x*2 – 6*x* + 5 ≤ 0    The curve  crosses the  *x*-axis at *x* = 1 and *x* = 5.    1  5  *x*  In set notation:  {*x* : 1 ≤ *x* ≤ 5} | First factorise the quadratic.  Sketch the curve and the *x*-axis.  The *x*2 term is  positive so the shape is  The solution is the set of values of *x* for which the curve is below the *x*-axis.  This set notation means ‘all the values of *x* so that *x* is between 1 and 5, including 1 and 5.’ |

**Note:** There are other correct ways of using set notation.   
In Example 2, you could instead have written  
 { *x* : 5 ≥ *x* ≥ 1}   
or { *x* : *x* ≤ 5} { *x* : *x* ≥ 1}  
You may agree that this last answer is more difficult to understand!

**Example 3** Solve .   
Give your answer using set notation.

|  |  |
| --- | --- |
| 9 – *x*2 < 0  (3 – *x*) (3 + *x*) < 0  The curve  crosses the *x*-axis at *x* = 3 and *x* = –3.    In set notation:  {*x* : *x* < –3}  { *x* : *x* > 3} | First factorise the quadratic.  Sketch the curve and the *x*-axis.  The *x*2 term is negative  so the shape is  The solution is the set of values of *x* for which the curve is below the *x*-axis.  This set notation means ‘the set of values of *x* that are less than –3 together with the set of values of *x* that are more than 3. |

**Example 4** Solve 4 – *x*2 < 3*x*.  
Give your answer using set notation.

|  |  |
| --- | --- |
| 4 – *x*2 < 3*x*        The curve  crosses the  *x*-axis at *x* = –4 and *x* = 1.    –4  1  *x*  {*x* : *x* < –4}  {*x* : *x* > 1} | Before you factorise the quadratic, you **must** rearrange to the form .  Multiply each term by –1.  This changes < to >.   Factorise the quadratic.  Sketch the curve and the *x*-axis.  The *x*2 term is  positive so the shape is  The solution is the set of values of *x* for which the curve is above the *x*-axis. |

**Note:** Instead of multiplying through by –1, you could factorise  and solve , which will give the same answer.

If you have a graphical calculator, you could draw *y* = 4 – *x*2 and *y* = 3*x* and write down the solution set.

Exercise

**1.** Solve these inequalities.   
 Give each answer using set notation.

**(a)**  **(b)** 

**(c)**  **(d)** 

**(e)**  **(f)** 

**(g)**  **(h)** 21 – *x* – 2*x*2 > 0

**(i)**  **(j)** 

**2.** Using set notation for each answer, find the values for *x* for which

**(a)** 2(*x* – 4) ≥ 3*x* + 1

**(b)** (3*x* + 7)(*x* – 4) ≥ 0

**(c)** both 2(*x* – 4) ≥ 3*x* + 2 **and** (3*x* + 7)(*x* – 4) ≥ 0

**3.** Using set notation for each answer, find the values for *x* for which

**(a)** 5*x* + 7 > 3*x* + 1

**(b)** 2*x*2 – 17*x* + 8 > 0

**(c)** both 5*x* + 7 > 3*x* + 1 **and** 2*x*2 – 17*x* + 8 > 0

**4.** For a real number *a*, the solution of is given by the set   
 {*x* : *x* < –6}  {*x* : *x* > 17}  
 Find the value of *a*.

**5.** The solution set of the equation 6*x*2 + 61*x* + *c* < 0 is {*x* : – < *x* < –}.   
 Find the value of *c*.

Answers

**1. (a)** {*x* : *x* > –1} **(b)** {*x* : *x* ≤ 5}

**(c)** {*x* : *x* < –2}  {*x* : *x* > 3} **(d)** {*x* : 0 < *x* < 4}

**(e)** {*x* : *x* ≤ –2}  {*x* : *x* ≥ –1} **(f)** {*x* : *x* < –1}  {*x* : *x* > –0.2}

**(g)** {*x* : –4 ≤ *x* ≤ 4} **(h)** {*x* : –3.5 < *x* < 3}

**(i)** {*x* : –2 < *x* < 3} **(j)** {*x* : *x* ≤ 2}  {*x* : *x* ≥ 6}

**2. (a)** {*x* : *x* ≤ –9}

**(b)** {*x* : *x* ≤ }  {*x* : *x* ≥ 4}

**(c)** {*x* : *x* ≤ –9} because **(a)**

–9

*x*

**(b)**

–

4

*x*

**both**

–9

*x*

**3. (a)** {*x* : *x* > –3}

**(b)** {*x* : *x* < }  {*x* : *x* > 8}

**(c)** {*x* : –3 < *x* < }  {*x* : *x* > 8} because **(a)**

–3

*x*

**(b)**



8

*x*

**both**



8

*x*

–3

**4.** 102 because the solution set is given by

–6

17

*x*

and the inequality therefore is or *x*2 – 11*x* – 102 > 0.

**5.** 85 because the solution set is given by





*x*

and the inequality therefore is  < 0

or which gives 