**2.7 Proportional relationships**

When two quantities *y* and *x* are **directly proportional** (), they both increase or decrease at the same rate. On a graph this is represented by a straight line through the origin; the equation of this line is *y = kx* for a constant *k*.

When two quantities *y* and *x* are **inversely proportional** (), when one quantity increases the other quantity decreases at the same rate. On a graph this is represented by a curve with equation  for a constant *k*.

There are other proportional relationships, for example

* *y* can be directly proportional to the square of *x* (), which is represented on a graph by the curve with equation *y = kx*2,
* *y* can be inversely proportional to the square root of *x* (), which is represented on a graph by the curve with equation .

Examples

**Example 1** The circumference of a circle is directly proportional to the diameter ().

**(a)** Sketch the graph of *C* against *D.*

When the diameter is 25.0 m the circumference is 78.5 m, both to 3 significant figures.

**(b)** Find the equation connecting *C* and *D*.

|  |  |
| --- | --- |
| **(a)**  *O*  *D*  *C*    **(b)** *C* = *kD*  78.5 = *k* × 25.0    So *C* = 3.14*D* | The graph is a straight line through the origin.  (You can also draw this with the *C*-axis horizontal and the *D*-axis vertical.)  Substitute *C* = 78.5 and *D* = 25.0  This is a well-known formula! |

**Example 2** The force of attraction, *F* newtons, between two spacecrafts is inversely proportional to the square of the distance, *d* metres, between them.

**(a)** Sketch the graph of *F* against *d.*

For two spacecrafts that are 2000 m away from each other, the force of attraction is 6.7 × 10–6 newtons.

**(b)** Find the equation connecting *F* and *d*.

|  |  |
| --- | --- |
| **(a)**    *O*  *d*  *F*  **(b)** | Both axes are asymptotes.  Substitute *F* = 6.7×10–6 and *d* = 2000 |

Exercise

**1.**  for positive values of *A* and *B*. *A* = 30 when *B* = 50.

**(a)** Find an equation connecting *A* and *B*.

**(b)** Find the value of *B* when *A* = 15. Give your answer to 1 decimal place.

**(c)** Sketch the graph of *A* against *B*.

**2.** *P* is inversely proportional to *Q* and *P* = 23 when *Q* = 10.

**(a)** Find an equation connecting *P* and *Q*.

**(b)** Find the value of *P* when *Q* = 1.85. Give your answer to 3 significant figures.

**(c)** For *Q* > 0,sketch the graph of *P* against *Q*.

**3.** The volume of a cylinder is directly proportional to the height ().

**(a)** Sketch the graph of *V* against *h*, for *h* > 0.

When the volume is 400 mm3 the height is 2 m.

**(b)** Find the equation connecting *V* and *h*.

**(c)** Use this equation to find the volume when the height is 3.5 m.

**4.** An object is travelling with velocity *v* m s–1. Its energy, *E* joules, is directly proportional to the square of *v*.

**(a)** Sketch the graph of *E* against *v*. for *v* > 0.

**(b)** *E* = 30 when *v* = 3. Find the equation connecting *E* and *v*.

**(c)** Use this equation to find, to 2 significant figures, the energy when the speed is 7.8 m s–1.

**5.** 

**(a)** Sketch the graph of *y* against *x* for positive values of *x* and *y*.

**(b)** *y* = 90 when *x* = 400. Find the equation connecting *y* and *x*.

**6.** 

**(a)** Sketch the graph of *y* against *x* for *x* > 0.

**(b)** *y* = 10 when *x* = 49. Find the equation connecting *y* and *x*.

Answers

**1. (a)**  **(b)** 

**(c)**

*O*

*B*

*A*

**2. (a)**  **(b)** When *Q* = 1.85, 

  **(c)**

*O*

*Q*

*P*

**3. (a) (b)** 2 m = 2000 mm

*O*

*h*

*V*



**(c)** 3.5 m = 3500 mm,   
   
 The volume is 700 mm3.

**4. (a) (b)** 

*O*

*v*

*E*

**(c)**   
 202.8 = 200 to 2 significant figures, the energy is 200 joules.

**5. (a) (b)** 

*O*

*x*

*y*

**6. (a) (b)** 

*O*

*x*

*y*