**6.1 and 6.2 The function of e*x* and its graph**

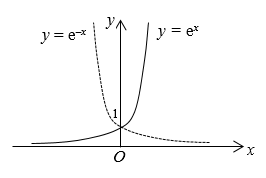
You are expected to know that

* (e*x*) = e*x*

This means: when you differentiate e*x* the result is e*x*.

* 
* if the rate of change of *y* is proportional to *y* then an exponential model should be used.

**Note:** the last bullet point is the same as: 



The graphs of *y* = e*x* and *y* = e*–x* are sketched on the diagram.

The curves are called exponential curves and pass through (0, 1).

*y* = e*x* is the only function where the gradient function is the same as the original function, i.e. both *y* = e*x*and .

Examples

**Example 1 (a)** Find the derivative of e5*x*.

**(b)** Find the gradient of *y* = e5*x* at the point .

|  |  |
| --- | --- |
| **(a)**  **(b)** Gradient = | ‘Find the derivative’ means ‘differentiate’.  Use  Substitute *x* = . You can simply use your calculator to calculate  in this question. |

**Example 2** The line *y* = 4*x* + *c* is a tangent to the curve *y* = e*x*.

Find the exact value of *c*.

|  |  |
| --- | --- |
| e*x* = 4 | The gradient of the curve equals the gradient of the tangent at the point where the lines meet.  Substitute *x* = ln 4 into *y* = e*x* to find the value of *y* at this point.  (ln 4, 4) is on thecurve and also on the line, so substitute into*y*= 4*x*+ *c* and solve to find *c*. |

**Example 3** *M* is the mass in grams of a substance *t* minutes after the start of an experiment.   
The rate of change of *M* is proportional to *M*. *M* = 20 when *t =* 0 and *M* = 22 when *t* = 1.

**(a)** Find an expression for *M* in terms of *t*.

**(b)** Find the amount of substance after 5 minutes.

|  |  |
| --- | --- |
| **(a)**        **(b)** When *t* = 5,  After 5 minutes there are 32.2 grams of the substance. | You must use an exponential model.  Substitute *M* = 20 and *t* = 0.  Substitute *M* = 22 and *t* = 1.  Give your answer in context. |

Exercise

**1.** Differentiate the following.

**(a)**  **(b)** 

**(c)**   **(d)** 

**2.** Find the gradient of the graph  when *x* = 3.

**3.** Sketch the following curves on the same diagram.

**(a)** *y* = 2*x* **(b)** *y* = 3–*x* **(c)** 

**4.** A curve has equation 

Find the coordinates of the point of the curve where the gradient of the tangent is 9.

**5.** Find the gradient of the graph  when *y* = 12.

**6.** The graph of  has a gradient equal to 3 when *y* = 5. Find the value of *k.*

**7.** *P* is the population of people on a small island *t* years after 2015.   
 The rate of change of *P* is proportional to *P*.   
 *P* = 100 initially and *P* = 150 when *t =* 5.

**(a)** Find an expression for *P* in terms of *t*.

**(b)** Find, to 3 significant figures, the number of people on the island after 50 years.

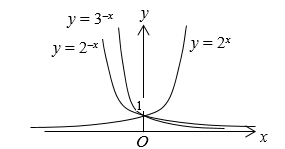
**(c)** Is this model realistic? Explain your answer.

Answers

**1. (a)**  **(b)**  **(c)**  **(d)** 

**2.** 

When *x* = 3, 

**3.** Note that 

And when *x* = 1, 2–1 > 3–1

**4.**    
 so  

You can also give the answer or  as you are not asked to give the answer in exact form.

so the point is 

**5.** 

**6.** When *y* = 5, e*kx* = 5



**7. (a)** 

When *t* = 0, 

When *t* = 5, 



**(b)** When *t* = 50,  = 5770 (to 3 significant figures)

After 50 years there are 5770 people on the island.

**(c)** No, eventually the island will be too small for the increasing number of people.