Section 2: Second order non-homogeneous equations

Exercise level 2

1. A particle of mass m kg moves in a straight line. Its position and velocity at time t s are x m and v ms^{-1} respectively. The particle experiences three forces:
    - $5mx$ N away from the origin
    - a resistance of $4mv$ N
    - a force $2m$ N in the positive direction.

   (i) Formulate the equation of motion of the particle as a second order differential equation for x.
   (ii) Find the general solution of the differential equation in part (i).
   (iii) Initially the particle is at rest 0.6 m from the origin in the positive direction. Find an expression for x at time t.

2. A particle P hangs on a spring which is attached to an oscillating point.
   The displacement of P from its initial position is modelled by the differential equation
   $$3\frac{\mathrm{d}^2x}{\mathrm{d}t^2}+4\frac{\mathrm{d}x}{\mathrm{d}t}+x=\cos t$$
   (i) Write down the auxiliary equation and find the complementary function for this equation.
   (ii) Find the particular integral for this equation and hence find the general solution.
   (iii) Show that the long-term behaviour of the particle is independent of the initial conditions.

3. A particle is at rest on a horizontal surface and is attached by a spring to a point which moves with constant acceleration towards the particle. The displacement x m of the particle from its initial position at time t s is modelled by the equation
   $$\frac{\mathrm{d}^2x}{\mathrm{d}t^2}+2\frac{\mathrm{d}x}{\mathrm{d}t}+10x=t^2$$
   (i) Find the general solution of the differential equation.
   (ii) Use the initial conditions to find an expression for x at time t.

4. A simple pendulum is modelled by the differential equation
   $$\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2}+k^2\theta=0$$
   where θ is the small angle that the pendulum makes with the vertical at time t s.
   (i) Find the general solution of this equation.

   An additional force is applied so that the model for θ becomes
   $$\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2}+k^2\theta=\cos kt$$
   (ii) Find the general solution of this differential equation.

   Initially the pendulum is at rest making an angle of α with the vertical.

   (iii) Find the solution of the differential equation with these initial conditions.
   (iv) Is this a good model for the behavior of the pendulum for large values of t