Section 2: The area of a sector

Exercise level 3 (Extension)

1. (a) Show that the graph of $r = \dfrac{1}{\sin\theta - \cos\theta}$ is a straight line, and find its equation.

   Show similarly that $r = \dfrac{1}{a\sin\theta - b\cos\theta}$ is a straight line, with equation $ay + bx = 1$.

   (b) Show also that $r = k\sec(\theta - \alpha)$, where $0 < \alpha < \frac{\pi}{2}$, is a straight line. If A is the foot of the perpendicular from O to this line, and B is the x-intercept of this line, find the area of triangle OAB
   (i) by direct calculation
   (ii) by integration using polar coordinates.

2. Sketch the curve $r = \theta + \dfrac{1}{\theta}$ for $0 < \theta < 4\pi$.

   Find the exact area of the central loop.
   Verify this with a spreadsheet or graphical program.