Section 2: The area of a sector

Exercise level 2

1. (i) Sketch the curve $r = 3 + 2\cos\theta$.
   (ii) Find the area enclosed by the curve.
   (iii) Find the equations of the tangents parallel to and perpendicular to the initial line. Give answers to three decimal places where necessary.

2. (i) Sketch the curve $r\cos\theta = a$, where $a > 0$, and give its Cartesian equation.
   (ii) If A is the area between the curve, the initial line, and $\theta = \alpha$, where $0 < \alpha < \frac{\pi}{2}$, then find A:
   (a) by using the formula $\frac{1}{2}ab\sin C$ for the area of a triangle
   (b) by using $A = \int \frac{1}{2}r^2 \mathrm{d}\theta$ with appropriate limits.

3. Suppose $r = \sin\theta - \cot\theta$
   (i) By considering $\frac{\mathrm{d}r}{\mathrm{d}\theta}$, show that r is positive and is increasing over the values $1 \le \theta \le \frac{\pi}{2}$.
   (ii) Find the exact area bounded by the curve and the rays $\theta = \frac{\pi}{3}$ and $\theta = \frac{\pi}{2}$.

4. (i) Sketch the curve $r = \cos 3\theta$ for $0 \le \theta < 2\pi$.
   (ii) The curve $r = \cos(3\theta + a)$, where $a > 0$, has the initial line as a line of symmetry. What is the smallest possible value for a?
   (iii) Sketch this curve.
   (iv) Find the total area enclosed by the curve.

5. (i) Find $\int \mathrm{cosec}\, x \,\mathrm{d}x$ by using the substitution $u = \cos x$.
   (ii) For the curve $r = 1 + \frac{1}{\sin\theta}$, find the area between the curve and the rays $\theta = \frac{\pi}{3}$ and $\theta = \frac{\pi}{2}$.

6. Sketch the curve $r = \frac{a}{\theta}$ for $\frac{\pi}{2} \le \theta \le 2\pi$. The area enclosed by this curve in quadrants 2, 3 and 4 is 1. Find the value of a.