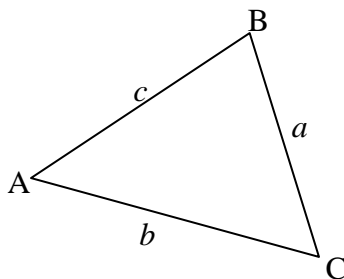


Section 1: Introducing the hyperbolic functions

Exercise level 3

1. (i) Prove that $\tanh x = \frac{e^{2x} - 1}{e^{2x} + 1}$.
 - (ii) If $y = a^2 \sinh x + b^2 \cosh x$, with $a^2 > b^2$, show that the curve has no turning points, but that it does cut the x -axis.
 - (iii) If instead $b^2 > a^2$, show that the curve does have a turning point, but does not cut the x -axis.
 - (iv) What happens if $a^2 = b^2$?
2. The diagram shows a triangle ABC, with side lengths a , b and c .



Given the equation $a \cosh x + b \sinh x = c$,

- (i) Show that if A is an obtuse angle, the equation has no real roots
 - (ii) Show that if A is a right angle, then there is one real root, $x = \ln\left(\frac{c}{a+b}\right)$
 - (iii) Show that if A is an acute angle, there are two real roots, and find them.
3. (i) Given $y = \sum_{n=0}^3 \sinh(nx) \cosh^n x$, find $\frac{dy}{dx}$ when $x = 0$.
 - (ii) Find $\sum_{n=0}^3 \int_0^1 (\sinh^n x + \cosh^n x) dx$.