Section 1: Introducing the hyperbolic functions

Exercise level 1

1. Express $\tanh 3x$ in terms of exponentials.

2. (i) Prove the identities $\sinh 2x = 2\cosh x \sinh x$ and $\cosh 2x = 1 + 2\sinh^2 x$.
   (ii) Hence express $\cosh x \sinh 4x$ in terms of $\sinh x$.

3. Differentiate $x^3 \cosh^2 4x$.

4. Use the identity $\cosh 2x = 1 + 2\sinh^2 x$ to find $\int \sinh^2 x \, dx$.

5. Solve the equation $4\cosh x + 5\sinh x = 6$.
   Give your answer in a logarithmic form.