Edexcel A level Maths Projectiles

Section 2: General equations

Exercise level 3 (Extension)

1. A particle is projected with speed u ms^{-1} on horizontal ground so that it reaches a height of more than 1.2 metres. Find the angle to the horizontal at which it must be projected if the difference in time between reaching 1.2 metres on the way up and down is 1 second and the distance apart of the points where it does so is 7 metres. (Take $g = 10$).

2. A particle is projected from the ground on horizontal terrain at a speed u ms^{-1} and inclination to the horizontal θ. Take the origin of coordinates as the point of projection, the x-axis as horizontal and the y-axis as vertical, both in the plane of the particle's motion.

   (i) Show that the equation of its trajectory is $y = x\tan\theta - \dfrac{gx^2}{2u^2\cos^2\theta}$

   By regarding this as a function of x and completing the square, show that the maximum height of the particle is $\dfrac{u^2\sin^2\theta}{2g}$. At what distance horizontally from the starting point is that maximum height achieved?

   (ii) Using the identity $\dfrac{1}{\cos^2\theta} \equiv 1+\tan^2\theta$, the equation of the trajectory can also be written as $y = x\tan\theta - \dfrac{gx^2}{2u^2}\left(1+\tan^2\theta\right)$

   Use this result to write down a quadratic equation which must be satisfied by $\tan\theta$ if the path passes through the point with coordinates $\left(x_0, y_0\right)$.

   If $x_0 \neq 0$, find conditions in terms of u and g for which

   (A) the equation has two distinct roots;
   (B) the equation has two equal roots;
   (C) the equation has no roots.

   Deduce that the point $\left(x_0, y_0\right)$ is *inaccessible* if $y_0 > \dfrac{u^2}{2g} - \dfrac{gx_0^2}{2u^2}$.

3. A particle P is projected over level ground with speed u and an angle θ above the horizontal. Derive an expression for the greatest height of the particle in terms of u, θ, and g.

   A particle is projected from the floor of a horizontal tunnel of height $\dfrac{9}{10}d$. Point P is $\tfrac{1}{2}d$ vertically and d metres horizontally along the tunnel from the point of projection. The particle passes through the point P and lands inside the tunnel without hitting the roof. Show that

   $$\arctan\frac{3}{5} < \theta < \arctan 3$$

   [2004 STEP Mathematics 1 no 9; this question appears by permission of Cambridge Assessment.]