Section 3: Sine and cosine rules

Notes and Examples

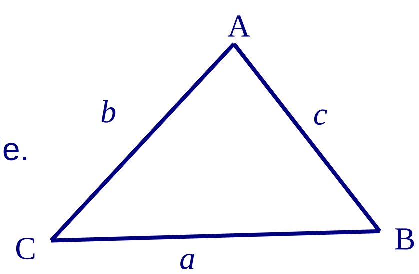
In this unit you learn about finding an unknown side or angle in any triangle. You will also learn a new formula for finding the area of a triangle.

These notes contain subsections on:

The sine rule

The sine rule:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



This form is easier to use when finding an unknown side.

The sine rule can also be written as:

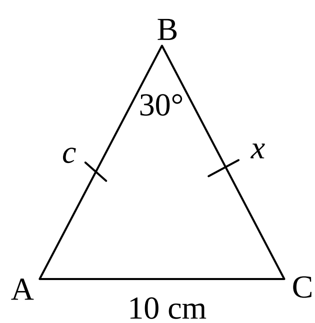
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

This form is easier to use when finding an unknown angle.

Note: When you use the sine rule to find a missing angle, θ, always check whether 180° - θ is a possible solution as well.

Example 1 shows a straightforward application of the sine rule to find an unknown side.

Example 1
Find the side BC in the triangle ABC.



Solution

The triangle is isosceles so $\angle \text{BAC}$ is $\dfrac{180°-30°}{2}=75°$

By the sine rule: $\dfrac{x}{\sin A}=\dfrac{b}{\sin B}$

So: $\dfrac{x}{\sin 75°}=\dfrac{10}{\sin 30°}$

$\Rightarrow x=\dfrac{10\sin 75°}{\sin 30°}$

so $x = 19.3$ cm (to 3 sig.fig.)

Example 2 shows a straightforward application of the sine rule to find an unknown angle.

Example 2

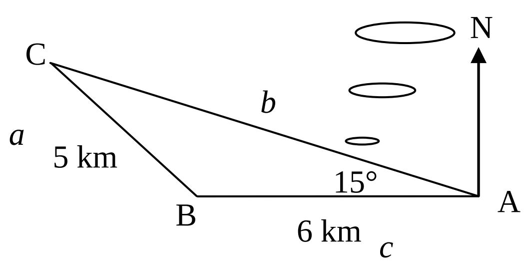
A, B and C are three points on a level plane. B is 6 km due west of A. C is 5 km from B and is on a bearing of 285° from A.

Find $\angle \text{ACB}$.

Solution

First draw a diagram:

Due west is a bearing of 270°, so this angle must be 15°.



Your diagram doesn't need to be accurate – just large enough to show all the information

By the sine rule: $\dfrac{\sin A}{a}=\dfrac{\sin C}{c}$

So: $\dfrac{\sin 15°}{5}=\dfrac{\sin C}{6}$

$\Rightarrow \quad \sin C=\dfrac{6\sin 15°}{5}$

$\sin C = 0.310....$

$C = 18.1°$ to 1 d.p.

Don't round here! Store the number in your calculator.

Check whether 180°- C is also a solution:

180°- 18.1° = 161.9° to 1 d.p.

This also works so $\angle \text{ACB}$ is 18.1° to 1 d.p. or 161.9° to 1 d.p

Angles A and C still add up to less than 180°

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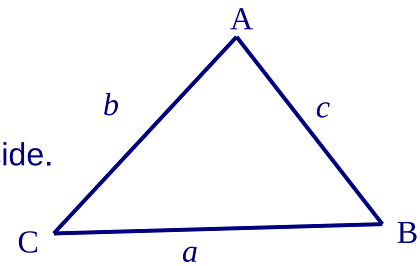
You can see examples similar to this using the Geogebra resource ***The sine rule – finding an angle***. This resource also shows geometrically what is happening when there is more than one possible solution.

The cosine rule



The cosine rule:

$$a^2 = b^2 + c^2 - 2bc\cos A$$

This form is easier to use when finding an unknown side.

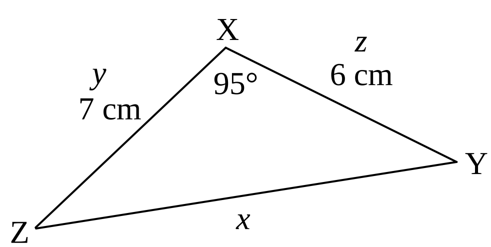
The cosine rule can also be written as:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

This form is easier to use when finding an unknown angle.

Example 3 shows an application of the cosine rule to find an unknown side.

Example 3
Find the side YZ in the triangle XYZ.



Solution

The cosine rule for this triangle is: $x^2 = y^2 + z^2 - 2yz\cos X$

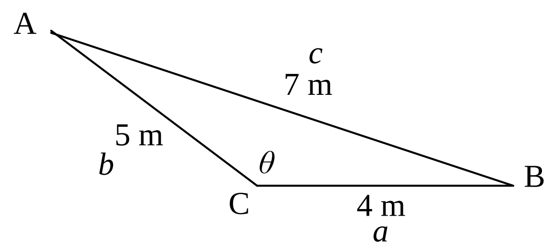
So: $x^2 = 7^2 + 6^2 - 2\times 7\times 6\cos 95°$

$x^2 = 92.32...$

$x = 9.61$ cm to 3 sig. fig.

Example 4 shows a straightforward application of the cosine rule to find an unknown angle.

Example 4
Find the angle θ in the triangle ABC.



Solution

The cosine rule for this triangle is: $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

$$\cos C = \frac{4^2 + 5^2 - 7^2}{2 \times 4 \times 5}$$

$$\cos C = -0.2$$

$$C = 101.5° \text{ to 1 d.p.}$$

You can see examples similar to this using the Geogebra resource ***The cosine rule – finding an angle***.

Choosing which rule to use

Use the sine rule when:

- you know 2 sides and 1 angle (not between the two sides) and want a 2nd angle (3rd angle is now obvious!)
- you know 2 angles and 1 side and want a 2nd side

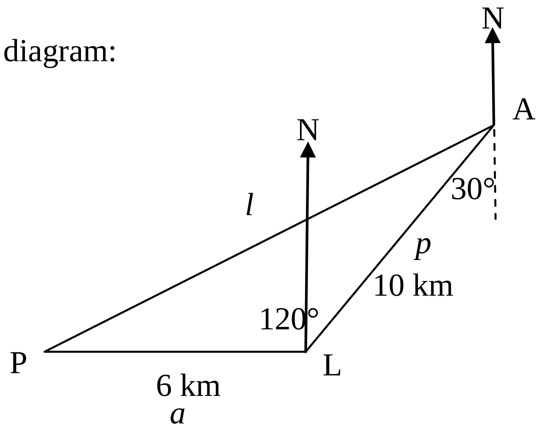
Use the cosine rule when:

- you know 3 sides and want any angle
- you know 2 sides and the angle between them and want the 3rd side

Example 5 shows how to decide whether to use the sine or the cosine rule.

Example 5
A ship sails from a port, P, 6 km due East to a lighthouse, L, 6 km away.
The ship then sails 10 km on a bearing of 030° to an island, A.
Find: (i) The distance AP
(ii) The bearing of P from A

Solution
First draw a diagram:



(i) You know 2 sides and the angle between them so you need the cosine rule.
The cosine rule for this triangle is:

$$l^2 = a^2 + p^2 - 2ap\cos l$$
$$l^2 = 6^2 + 10^2 - 2\times 6\times 10\cos 120°$$
$$l^2 = 196$$
$$l = 14 \text{ km}$$

So the distance AP is 14 km.

(ii) You can now use either the cosine rule or the sine rule to find the angle PAL.
The sine rule for this triangle is: $\frac{\sin A}{a} = \frac{\sin L}{l}$

So:
$$\frac{\sin A}{6} = \frac{\sin 120°}{14}$$
$$\therefore \sin A = \frac{6\sin 120°}{14}$$
$$\therefore \sin A = 0.371....$$
$$A = 21.8°$$

So the bearing is 180° + 30° + 21.8° = 231.8° to 1 d.p.

Check whether 180° - 21.8°= 158.2° is also a solution.
It isn't because the angles in the triangle would total more than 180°.

The area of a triangle

To find the area of any triangle you can use the rule:

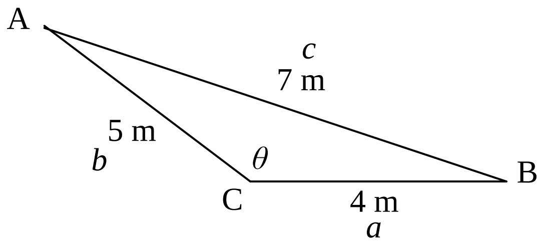
$$\text{Area of triangle ABC} = \frac{1}{2}ab\sin C$$

So you need two sides and the angle between them.

Example 6 shows how to use this formula.

Example 6
Find the area of triangle ABC from Example 4.

Solution



In Example 4, angle C was found to be 101.5° to 1 d.p.

Using the formula Area = $\frac{1}{2}ab\sin C$ gives:

Area of triangle ABC = $\frac{1}{2}\times 4\times 5\times \sin 101.5...° = 9.80\,\text{m}^2$